Example 1. $y' = -2x$.

Example 2. Verify that $y = ce^{2x}$ (c is any real number) are solutions for the differential equation $y' = 2y$.

Example 3. One model for the growth of a population

$$\frac{dP}{dt} = kP$$

where k is a constant number, t is the time (independent variable) and P is the number of individuals in the population (dependent variable).

Example 4. Verify that $y = a\sin 3x + b\cos 3x$ (where, a, b are any real numbers) are solutions for the differential equation $y'' = -9y$.

Example 5. (1) For what values of r does the function $y = e^{rx}$ a solution for $y'' - y' - 2y = 0$?

(2) If r_1 and r_2 are the values of r that you found in part (1), show that every member of the family of functions $y = ae^{r_1 x} + be^{r_2 x}$ is a solution for $y'' - y' - 2y = 0$.

Example 6. (1) For what values of r does the function $y = e^{x^r}$ a solution for $y' = 3x^2 y$?

(2) Show that every member of the family of functions $y = ke^{x^3}$ is a solution for $y' = 3x^2 y$. Here, k is any real number.

(3) Find a solution of differential equation $y' = 3x^2 y$ with initial condition $y(0) = 2$.

Example 7. [Pierre-Francois Verhulst, 1840s] The world population growth is modeled by the differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

where M is the carrying capacity.

Let us see what can we obtain from this model.