

Find the Maclaurin series or Taylor series for the following functions.

Example 1-4

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}. \quad \text{The radius of convergence } R = \infty.$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}. \quad \text{The radius of convergence } R = \infty.$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}. \quad \text{The radius of convergence } R = \infty.$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n. \quad \text{The radius of convergence } R = 1.$$

Example 5. $f(x) = xe^{3x}$

Example 6. $f(x) = x^4$ at $a = 1$

Example 7. $f(x) = \sin \pi x$

Example 8. $f(x) = \ln x$ at $a = 1$.

Show series converges and **Find the sum.**

Example 9. $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

Example 10. $\sum_{n=0}^{\infty} (-1)^n \frac{(\pi/3)^{2n}}{(2n)!}$

Example 11. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n+1}}{3^{2n+1}(2n+1)!}$

Example 12. $\sum_{n=0}^{\infty} \frac{(\ln 5)^n}{n!}$

Example 13. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^n}{n}$

Example 14. $\sum_{n=1}^{\infty} \frac{(-2)^n}{n 3^n}$