

**Example 1.** Find a formula for the general term of the sequence

$$\left\{ \frac{4}{3}, -\frac{5}{9}, \frac{6}{27}, -\frac{7}{81}, \frac{8}{243}, \dots \right\}$$

**Example 2.** Some hard sequences with no formulas.

(1) The sequence  $\{d_n\}$ , where  $d_n$  is the  $n$ -th decimal of  $\pi$ .

(2) The sequence  $\{p_n\}$ , where  $p_n$  is the  $n$ -th prime number.

(3) **Fibonacci sequence**  $\{f_n\}$  is defined recursively by the conditions

$$f_1 = 1, \quad f_2 = 1, \quad f_n = f_{n-1} + f_{n-2} \quad \text{for } n \geq 3.$$

**Example 3.** Plot the sequence  $\{a_n\}$ , where  $a_n = \frac{n}{n+1}$ .

**Example 4.**  $\lim_{n \rightarrow \infty} 1/n = 0$ .

**Example 5.** The sequence  $\{(-1)^n\}$  diverges.

**Example 6.**  $\lim_{n \rightarrow \infty} \frac{1}{n^p} = 0$ , if  $p > 0$ . (How about if  $p \leq 0$ ?)

**Example 7.**  $\lim_{n \rightarrow \infty} r^n = 0$ , if  $|r| < 1$ . (How about if  $|r| \geq 1$ ?)

8. Find  $\lim_{n \rightarrow \infty} \frac{5n}{n+1}$

14. Find  $\lim_{n \rightarrow \infty} \frac{\sin n}{2n}$ .

9. Find  $\lim_{n \rightarrow \infty} \frac{\ln n}{2n}$

15. Find  $\lim_{n \rightarrow \infty} 2ne^{-n}$ .

10. Find  $\lim_{n \rightarrow \infty} \cos(\pi/n)$ .

16. Find  $\lim_{n \rightarrow \infty} \frac{2n+10}{n^2-4}$ .

11. Find  $\lim_{n \rightarrow \infty} \tan(2^{-n})$ .

17. Find  $\lim_{n \rightarrow \infty} 2 - \frac{2^n}{3^{n-1}}$ .

12. Find  $\lim_{n \rightarrow \infty} \frac{n}{\sqrt{3n^2+2}}$ .

18. Find  $\lim_{n \rightarrow \infty} \frac{n!}{n^n}$ .

13. Find  $\lim_{n \rightarrow \infty} \frac{(-1)^n n}{\sqrt{3n^2+2}}$ .

**Example\* 19** Find the limit of the sequence  $\{a_n\}$  defined by the recurrence relation  $a_1 = 2$  and  $a_{n+1} = \frac{1}{2}(a_n + 6)$  for  $n = 1, 2, 3, \dots$