- Math 182 – Calculus 2 (Spring 2019)
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1. About the Syllabus:

Read the Syllabus carefully!

2. Review some backgrounds:

• Numbers:

Natural numbers $\mathbb{N}$
Integers $\mathbb{Z}$
Rational numbers $\mathbb{Q}$
Real numbers $\mathbb{R}$
Complex numbers $\mathbb{C}$

• Functions:

Single variable functions:

$f(x) = 2x + 3$; $f(x) = x^2 + 8$; $f(x) = 2e^x + 3$; $f(x) = 2\sin(x)$; $f(x) = \ln(x) + 3x - 2$; ...

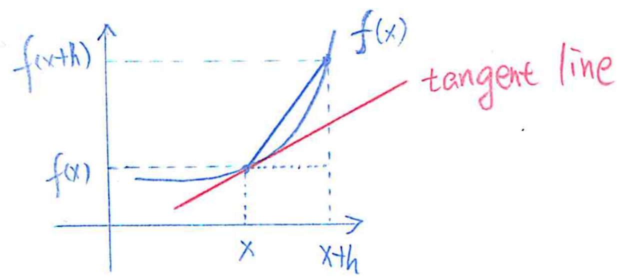
3. Student Learning Outcomes of Calculus 1:

Upon completion of this course, students will be able to:

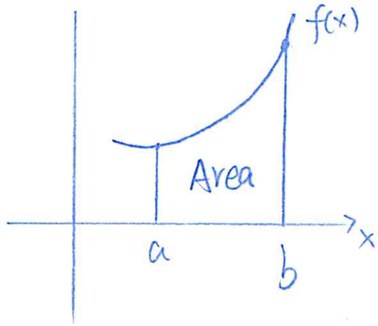
$\lim_{x \to a} f(x)$

1. Demonstrate an understanding of concepts and the terminology of **limits** through applications and examples.
2. Compute the **derivative** of a function using the definition, rules of differentiation, slopes of tangent lines and describe it as rate of change in natural and physical phenomena.
3. Compute basic **integrals** using Riemann sums as well as the Fundamental Theorem of Calculus.

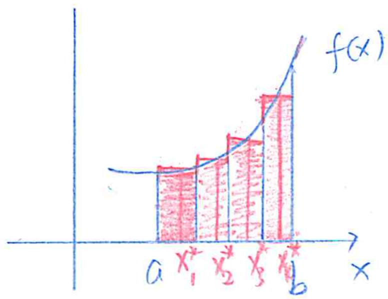
$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$



5.1. Area and Estimating with Finite (Riemann) Sums



$\Delta x = \frac{b-a}{4}$



Area ≈ sum of Area of the 4 rectangles.

$= f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x + f(x_4^*)\Delta x$

5.2. The Definite Integral

Let $f(x)$ be a continuous function defined on the interval $[a, b]$. The **definite integral** (accumulated change) of $f(x)$ from a to b is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

$\Delta x = \frac{b-a}{n}$

Some properties:

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

5.3. The Fundamental Theorem of Calculus.

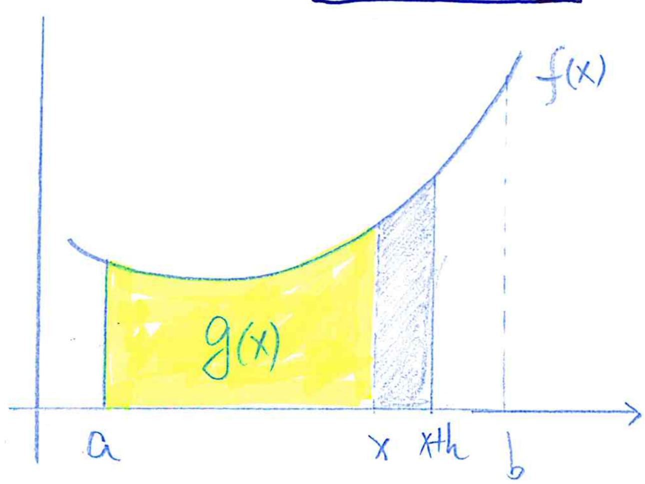
Theorem 1. If $f(x)$ is a continuous function on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a).$$

Theorem 2. If $f(x)$ is a continuous function on the interval $[a, b]$, then

$$g(x) = \int_a^x f(t)dt$$

is continuous and $\boxed{g'(x) = f(x).}$



$$f(x) \simeq \frac{g(x+h) - g(x)}{h}$$

$$f(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = g'(x)$$

Review of some formulas from Calculus 1

Derivatives:

Function $f(x)$	Derivative $f'(x)$
x^n	$n \cdot x^{n-1}$
b^x	$(\ln b) \cdot b^x$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\sin(kx)$	$k\cos(kx)$
$\cos(kx)$	$-k\sin(kx)$

Indefinite integral:

Function $f(x)$	$\int f(x)\, dx$
x^n , $(n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
b^x	$\frac{b^x}{\ln(b)} + C$
e^{kx}	$\frac{e^{kx}}{k} + C$
$\sin(kx)$	$-\frac{1}{k}\cos(kx) + C$
$\cos(kx)$	$\frac{1}{k}\sin(kx) + C$

Chain Rule: $\big(f(g(x))\big)' = f'(g(x)) \cdot g'(x)$

Product Rule: $\big(f(x)g(x)\big)' = f'(x)g(x) + f(x)g'(x)$

Evaluate the definite integrals:

Example 1. Find $\displaystyle\int_1^2 \frac{4x^5+6}{x^2}\,dx$

$$\int_1^2 \frac{4x^5+6}{x^2}\,dx = \int_1^2 4x^3+6x^{-2}\,dx$$
$$= x^4-6x^{-1}\Big|_1^2$$
$$= (2^4-6(\tfrac{1}{2}))-(1-6) = 13+5 = 18$$

Example 2. Find $\displaystyle\int_0^\pi \sin 3x\,dx$

$$\int_0^\pi \sin 3x\,dx = -\frac{1}{3}\cos 3x\Big|_0^\pi$$
$$= -\frac{1}{3}(\cos 3\pi - \cos 0)$$
$$= \frac{2}{3}$$

Example 3. Find $\displaystyle\int_0^1 e^{2x}\,dx$

$$\int_0^1 e^{2x}dx = \frac{1}{2}e^{2x}\Big|_0^1 = \frac{1}{2}e^2-\frac{1}{2}$$

Example 4 Show that $\ln x = \int_1^x \frac{1}{t}\,dt$

$$\int_1^x \frac{1}{t}\,dt = \ln t\Big|_1^x = \ln x - \ln 1 = \ln x$$

Find the derivative of the following functions.

Example 5. $f(x) = \int_3^{2x^3-1} \sin(2t)\,dt$

Suppose $F(t)$ is an antiderivative of $\sin 2t$, then $F'(t) = \sin 2t$

$$f(x) = F(2x^3-1) - F(3)$$

$$f'(x) = F'(2x^3-1)\cdot 6x^2 - 0 = \left(\sin 2(2x^3-1)\right)\cdot 6x^2$$

Example 6. $f(x) = \int_1^{x^2} e^{t^2}\,dt$

Suppose $F(t)$ is an antiderivative of e^{t^2}, then $F'(t) = e^{t^2}$

Then $f(x) = F(x^2) - F(1)$

$$f'(x) = F'(x^2)\cdot 2x - 0$$

$$= e^{(x^2)^2}\cdot 2x = e^{x^4}\cdot 2x$$