

Two important examples.

1. Geometric Series.

(1.) If $|r| < 1$, the geometric series $\sum ar^{n-1}$ is convergent and

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r}.$$

(2.) If $|r| \geq 1$, then the geometric series $\sum ar^{n-1}$ is divergent.

2. p-series

(1.) If $p > 1$, the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent.

(2.) If $p \leq 1$, the p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent.

Given a series $\sum a_n$, we have several tests. Which one does one use?

1. Divergence Test. (First try.)

2. Integral Test. For a nice function $f(x)$ such that $a_n = f(n)$.

▷ For Positive Series

3. Comparison Test.

4. Limit Comparison Test.

▷ For Alternating Series:

5. Test for alternating series.

▷ Absolutely Convergent

6. Ratio Test (With form r^n or $n!$)

7. Root Test*