§9.3 Separable Equations

A differential equation of the form

$$\frac{dy}{dx} = f(x)g(y)$$

is called a **separable equation**.

If $g(y_0) = 0$, then $y = y_0$ is an *equilibrium solution*.

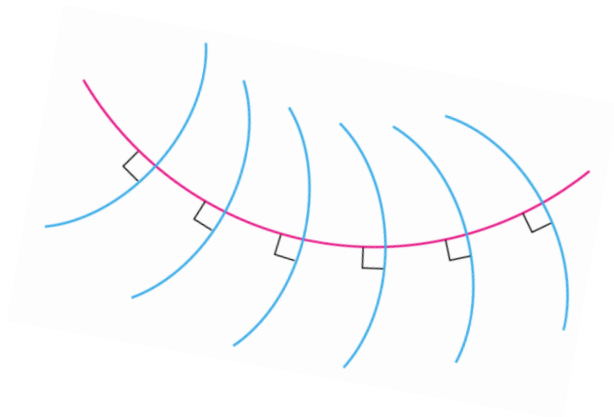If $g(y) \neq 0$, we can write the equation as

$$\frac{dy}{g(y)} = f(x)dx.$$
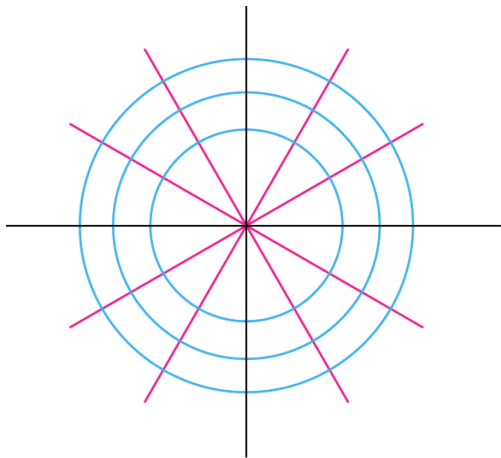
Integrate both sides, we have

$$\int \frac{dy}{g(y)} = \int f(x)dx.$$

We can solve y from the last equation.

An **orthogonal trajectory** of a family of curves is a curve that intersects each curve of the family orthogonally (or perpendicularly, or at right angles).

Example 6. Find the orthogonal trajectories of the family of curves $y = kx$ for $k \in \mathbb{R}$.

Example 7. Find the orthogonal trajectories of the family of curves $y = kx^2$ for $k \in \mathbb{R}$.