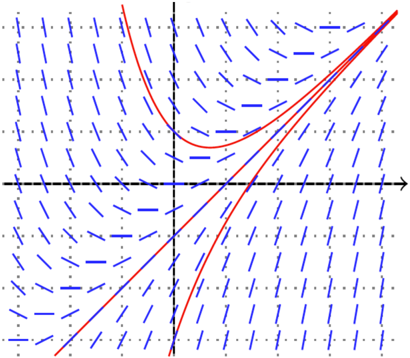
§9.2 Slope Fields and Euler's Method

1. Graphical approach (Slope fields, or direction fields)

If $y = f(x)$ is a solution for the differential equation $\dfrac{dy}{dx} = F(x, y)$, then the *slope of curve* $y = f(x)$ at (x, y) is $F(x, y)$.

At each point (x, y) draw a line segment with slope $F(x, y)$. The solution is the curve tangent at this point.

Example 1. $\dfrac{dy}{dx} = x - y$ (Use slope fields).



2. Numerical approach (Euler's Method)

Find approximating solution for the differential equation $\frac{dy}{dx} = F(x, y)$ with initial value $y(x_0) = y_0$ using *Euler's Method with step size h*:

(1). Set step size h; (the smaller h, the better estimation.)
(2). Start with point (x_0, y_0);
(3). Define a sequence $x_n = x_{n-1} + h$;
(4). Then y_n is computed by the sequence

$$y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$$

Example 2. Use Euler's method with step size $h = 0.5$ to solve $\dfrac{dy}{dx} = x - y$ with initial value $y(0) = 1$.