§9.1 Modeling with Differential Equations

- We have an introduction about differential equations in this chapter. There will be a class **Math 285**–Differential Equations after the class **Math 283**–Calculus 3.

- To solve some real world problems, we need to set up an equation. Some times, we need differential equations.
- A **differential equation** is an equation containing an *unknown function* $f(x)$ (or denoted by y) and some of its *derivatives* y', y'', ...

Example (1)

$y' = -2x$.

- A differential equation often has (infinitely) many solutions.
- Solving an equation is hard in general, but verifying whether or not a function $f(x)$ is a solution is easy.

Example (3)

One model for the growth of a population

$$\frac{dP}{dt} = kP$$

where k is a constant number, t is the time (independent variable) and P is the number of individuals in the population (dependent variable).

- When we apply differential equations to real world problems, we are interested in finding a particular solution satisfying a condition of the form $y(t_0) = y_0$, which is called an **initial condition**.
- This kind differential equation with initial condition is called **initial-value problem**.

Many populations start by increasing in an exponential model, however, the populations decrease when they approach its **carrying capacity** M.

Example 7. [Pierre-Francois Verhulst, 1840s] The world population growth is modeled by the differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

where M is the carrying capacity.

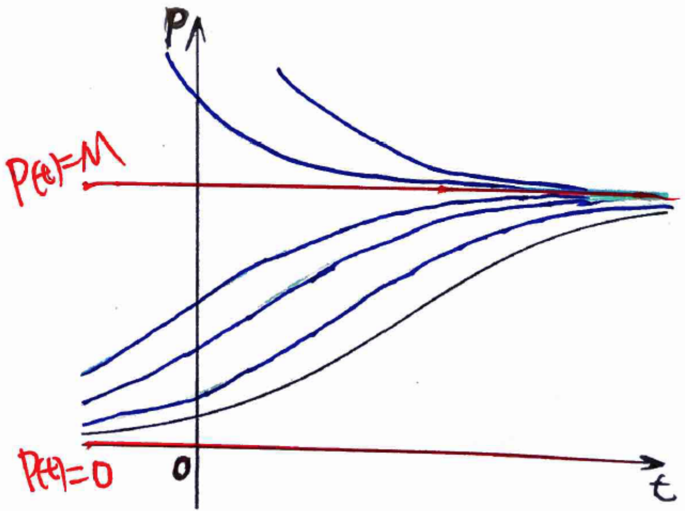
Let us see what can we obtain from this model.

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

1. The constant functions $P(t) = 0$ and $P(t) = M$ are solutions for the differential equation, which are called *equilibrium solutions*.

2. If the initial population $P(0) < M$, then the right side of the equation is positive, hence $\frac{dP}{dt} > 0$ and the population increases.

3. If the initial population $P(0) > M$, then the right side of the equation is negative, hence $\frac{dP}{dt} < 0$ and the population decreases.



P
P(t)=M
P(t)=0
0
t