§6.1 Areas Between Curves

We can use definite integral $\int_a^b f(x)\,dx$ to calculate area of region under the graph of a **positive** function $f(x)$ and above the x-axis.

Now, we use integral to calculate area of region that lies between the graphs of two continuous functions $f(x)$ and $g(x)$. If $f(x) \geq g(x)$ in the interval $[a, b]$, we can use the following formula to calculate the area.

Areas Between Curves

If $f(x) \geq g(x)$ in the interval $[a, b]$, the **area** A of the region bounded by the curves $f(x)$, $g(x)$ and the lines $x = a$, $x = b$ can be computed by

$$A = \int_a^b \left(f(x) - g(x)\right)\,dx$$

§6.1 Areas Between Curves

If we don't have the assumption $f(x) \geq g(x)$, how should we do?

Areas Between Curves

The **area** A of the region bounded by the curves $f(x)$, $g(x)$ and the lines $x = a$, $x = b$ can be computed by

$$A = \int_a^b |f(x) - g(x)| \, dx$$

Here $|f(x) - g(x)|$ is the absolute value defined by

$$|f(x) - g(x)| = \begin{cases} f(x) - g(x) & \text{when } f(x) \geq g(x) \\ g(x) - f(x) & \text{when } f(x) \leq g(x) \end{cases}$$

Regarding x as a function of y

Same principle holds for finding areas between $x = f(y)$ and $y = g(y)$.

Areas Between Curves

If $f(y) \geq g(y)$ in the interval $[c, d]$, the **area** A of the region bounded by the curves $f(y)$, $g(y)$ and the lines $y = c$, $y = d$ can be computed by

$$A = \int_c^d [f(y) - g(y)]\, dy$$