§5.5 Substitution Rule (*u*-substitution)

Recall the Chain Rule for derivative of the composition F(u(x)):

$$(F(u(x)))' = F'(u(x))u'(x).$$

Take indefinite integral for both sides of the chain rule formula.

$$\int (F(u(x)))' dx = \int F'(u(x))u'(x) dx.$$

Suppose F'(u) = f(u). Simplify both sides,

$$F(u(x)) + C = \int F'(u(x))u'(x) \ dx.$$

u-substitution

Then, we have the Substitution Rule for indefinite integral:

$$\int f(u(x))u'(x) \, dx = F(u(x)) + C$$

where F'(u) = f(u).