5.2. The Definite Integral

Let $f(x)$ be a continuous function defined on the interval $[a, b]$. The **definite integral** (accumulated change) of $f(x)$ from a to b is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

Some properties:

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

5.3. The Fundamental Theorem of Calculus.

Theorem (FTC 1)

If $f(x)$ is a continuous function on the interval $[a, b]$ and $F(x)$ is any antiderivative of $f(x)$, then

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a).$$

Theorem (FTC 2)

If $f(x)$ is a continuous function on the interval $[a, b]$, then

$$g(x) = \int_a^x f(t)dt$$

is continuous and $g'(x) = f(x)$.

Review of some formulas from Calculus 1

Derivatives:

Function $f(x)$	Derivative $f'(x)$
x^n	$n \cdot x^{n-1}$
b^x	$(\ln b) \cdot b^x$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\sin(kx)$	$k\cos(kx)$
$\cos(kx)$	$-k\sin(kx)$

Indefinite integral:

Function $f(x)$	$\int f(x)\ dx$
x^n , $(n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
b^x	$\frac{b^x}{\ln(b)} + C$
e^{kx}	$\frac{e^{kx}}{k} + C$
$\sin(kx)$	$-\frac{1}{k}\cos(kx) + C$
$\cos(kx)$	$\frac{1}{k}\sin(kx) + C$