§11.9 Representations of Functions as Power Series

Recall that the Geometric Series

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$$

converges when $|x| < 1$, and we have

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$$

More Generally, we want to express a function $f(x)$ as a power series:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n, \qquad \text{for } x \in I.$$

Theorem

If a function $f(x)$ can be written a power series:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n, \qquad \mathit{for}\ |x-a| < R,$$

*then, the **derivative** of $f(x)$ is*

$$f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$$

*and the **indefinite integral** of $f(x)$ is*

$$\int f(x)dx = \sum_{n=0}^{\infty} \frac{c_n (x-a)^{n+1}}{n+1} + C$$