§11.8 Power Series

Definition

More generally, a **power series** is

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

where $c_n \in \mathbb{R}$ are constant numbers called coefficients and x is a variable.

A power series may converge for some $x \in \mathbb{R}$ of and diverge for some other $x \in \mathbb{R}$.

Definition

More generally, a **power series centered at a** is

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + c_3(x-a)^3 + \cdots$$

Theorem

For a power series $\sum_{n=0}^{\infty} c_n(x-a)^n$, *there are only 3 possibilities:*

(1) *The series converges only when* $x = a$

(2) *The series converges for all* x.

(3) *There is a number* $R > 0$ *such that the series converges when* $|x-a| < R$ *and diverges when* $|x-a| > R$.

The number R in case (3) is called the **radius of convergence** of the power series.

The set of all x for which series converges is called the **interval of convergence**.