§11.5 The Alternating Series

Definition

Let $\sum b_n$ be a positive series, that is $b_n > 0$ for all n.
Then $\sum(-1)^n b_n$ or $\sum(-1)^{n+1} b_n$ is called an **alternating series**.

Theorem (Alternating Series Test)

*Let $\sum b_n$ be a **positive** series. If b_n is **decreasing** and $\lim_{n\to\infty} b_n = 0$, then the alternating series*

$$\sum(-1)^{n+1} b_n = b_1 - b_2 + b_3 - b_4 + b_5 - b_6 + \cdots$$

*is **convergent**.*

Remainder Estimate for Alternating Series*

Let $\sum b_n$ be a **positive, decreasing** series such that $\lim_{n\to\infty} b_n = 0$.

Theorem (Remainder Estimate for Alternating Series*)

Let $s = \sum b_n$ and $s_n = \sum_{k=1}^{n} b_k$.
If we denote the remainder of the sequence as $R_n = s - s_n$, then

$$|R_n| = |s - s_n| \leq b_{n+1}$$