§11.4 The Comparison Tests

Theorem (The Comparison Test)

Suppose both $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ are **positive** series.

1. If $a_n \leq b_n$ for all n, and $\sum b_n$ is convergent, then $\sum a_n$ is convergent.
2. If $a_n \geq b_n$ for all n, and $\sum b_n$ is divergent, then $\sum a_n$ is divergent.

The Limit Comparison Test

Theorem (The Limit Comparison Test)

Suppose both $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ are **positive** series.
If the limit

$$\lim_{n \to \infty} \frac{a_n}{b_n} = c$$

is a positive finite number, **then**
    $\sum a_n$ is convergent if and only if $\sum b_n$ is convergent.