Review: §11.2 Series

Example

Example 3 (Geometric Series $\sum ar^{n-1}$). The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

is convergent if $|r| < 1$ and

$$\sum ar^{n-1} = \frac{a}{1-r} \qquad \text{if } |r| < 1.$$

If $|r| \geq 1$, then the geometric series $\sum ar^{n-1}$ is divergent.

Theorem (Divergence Test)

If $\lim_{n\to\infty} a_n \neq 0$, *then the series* $\sum a_n$ *is divergent.*

Warning:

If $\lim_{n\to\infty} a_n = 0$, we know **nothing**!

Example (Harmonic Series)

The Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, although $\lim_{n\to\infty} \frac{1}{n} = 0$.

(How to know?)

Example . Is the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent or divergent?

Example . Is the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ convergent or divergent?

§11.3 The Integral Test and Estimates of Sums

Suppose $f(x)$ is **continuous**, **positive** and **decreasing** on $[1, \infty]$ and $a_n = f(n)$.

Theorem (The Integral Test)

The series $\sum_{n=1}^{\infty} a_n$ *is convergent if and only if the improper integral* $\int_1^{\infty} f(x)\, dx$ *is convergent.*

Warning: $\sum_{n=1}^{\infty} a_n$ and $\int_1^{\infty} f(x)\, dx$ are convergent to different values!

Example (p-series)

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$.

The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is divergent if $p \leq 1$.