§11.10 Taylor and Maclaurin Series

Theorem (Taylor Series)

Suppose a function $f(x)$ can be written a power series:

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n, \qquad for\ |x-a| < R,$$

Then

$$c_n = \frac{f^{(n)}(a)}{n!}$$

This is called **Taylor series** of $f(x)$ at $x = a$.

It is called **Maclaurin series** if $a = 0$.

Theorem

If a function $f(x)$ can be written a power series:

$$f(x) = \sum_{n=1}^{\infty} c_n (x-a)^n, \qquad for\ |x-a| < R,$$

*then, the **derivative** of $f(x)$ is*

$$f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$$

*and the **indefinite integral** of $f(x)$ is*

$$\int_a^x f(x)dx = \sum_{n=1}^{\infty} \frac{c_n (x-a)^{n+1}}{n+1} + C$$

Example

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

The radius of convergence $R = \infty$.

Example

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

The radius of convergence $R = \infty$.

Example

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

The radius of convergence $R = \infty$.

Example

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$$

The radius of convergence $R = 1$.

Example

$$\ln(x+1) = \sum_{n=0}^{\infty} \frac{(-1)^{n-1} x^n}{n}$$

The radius of convergence $R = 1$.