

Name: Key

Quiz No. 9
Sections 1202, 1203
4/11/19

1. (5 pts.) Show $y = -x \cos x - x$ is a solution of the following initial value problem.

$$xy' - y = x^2 \sin x, y(\pi) = 0$$

$$y(\pi) = -(\pi)\cos(\pi) - (\pi) = 0 \checkmark$$

$$y = -x \cos x - x; \quad y' = -\cos x + x \sin x - 1$$

$$xy' - y = x^2 \sin x$$

$$\begin{aligned} x(-\cos x + x \sin x - 1) - (-x \cos x - x) &= x^2 \sin x \\ -x \cos x + x^2 \sin x - x + x \cos x + x &= x^2 \sin x \\ x^2 \sin x &= x^2 \sin x \\ \text{LHS} &= \text{RHS} \checkmark \end{aligned}$$

2. (5 pts.) Find the solution of the differential equation that satisfies the given initial condition.

$$y' = \frac{\sin x}{y}, y(0) = -1$$

$$\frac{dy}{dx} = \frac{\sin x}{y}$$

$$\int y \, dy = \int \sin x \, dx$$

$$\frac{1}{2} y^2 = -\cos x + C$$

$$\frac{1}{2} (-1)^2 = -\cos(0) + C$$

$$\frac{1}{2} = -1 + C$$

$$C = \frac{3}{2}$$

$$\frac{1}{2} y^2 = -\cos x + \frac{3}{2}$$

$$y^2 = -2\cos x + 3$$

$$y = \pm \sqrt{-2\cos x + 3}$$

$$y(0) = -1$$

$$\Rightarrow \boxed{y = -\sqrt{-2\cos x + 3}}$$