Name: Solutions

Quiz No. 5
Sections 1202,1203
3/7/19

1. (5 pts.) Determine whether the series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} 2^n(-5)^{-n}$$

$$a_n = 2^n(-5)^{-n} = \frac{2^n}{(-5)^n} = \left(-\frac{2}{5}\right)^n = \left(-\frac{2}{5}\right)\left(-\frac{2}{5}\right)^{n-1}$$

So $\sum_{n=1}^{\infty} 2^n(-5)^{-n}$ is a geometric series with $a = -\frac{2}{5}$ and $r = -\frac{2}{5}$

$|r| = \frac{2}{5} < 1$. So the series is convergent.

The sum is $\sum_{n=1}^{\infty} 2^n(-5)^{-n} = \left(-\frac{2}{5}\right)\frac{1}{1+\frac{2}{5}} = -\frac{2}{7}$

2. (5 pts.) Determine whether the series converges or diverges. If it converges, find its sum.

$$\sum_{n=1}^{\infty} \frac{n^4}{6n^5+n^2+5}$$

$$\frac{a_n}{b_n} = \frac{\frac{n^4}{6n^5+n^2+5}}{\frac{1}{n}} = \frac{n^4}{6n^5+n^2+5}\cdot\frac{n}{1} = \frac{n^5}{6n^5+n^2+5} = \frac{1}{6+\frac{1}{n^3}+\frac{5}{n^5}}$$

So $\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{1}{6}$

Since $\sum b_n = \sum \frac{1}{n}$ is divergent, by limit comparison test

$\sum a_n = \sum_{n=1}^{\infty} \frac{n^4}{6n^5+n^2+5}$ is divergent.