University of Nevada, Reno — MATH182 (Calculus 2)
Midterm 3. Spring 2019 — Version

Solutions

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To receive full credit for a problem you must show all necessary work.

1. (10 points) Use the arc length formula to find the length of the curve

$$y = 3(2x-1)^{3/2}$$

for $1 \leq x \leq 3$.

$$y' = \frac{9}{2}(2x-1)^{\frac{1}{2}} \cdot 2 = 9(2x-1)^{\frac{1}{2}}$$

By arc length formula

$$L = \int_1^3 \sqrt{1+(y')^2}\, dx = \int_1^3 \sqrt{1+81(2x-1)}\, dx$$

$$= \int_1^3 \sqrt{162x-80}\, dx$$

$u = 162x - 80$

$du = 162\, dx$

$dx = \frac{1}{162} du$

$$= \int_{82}^{406} u^{\frac{1}{2}} \frac{1}{162} du$$

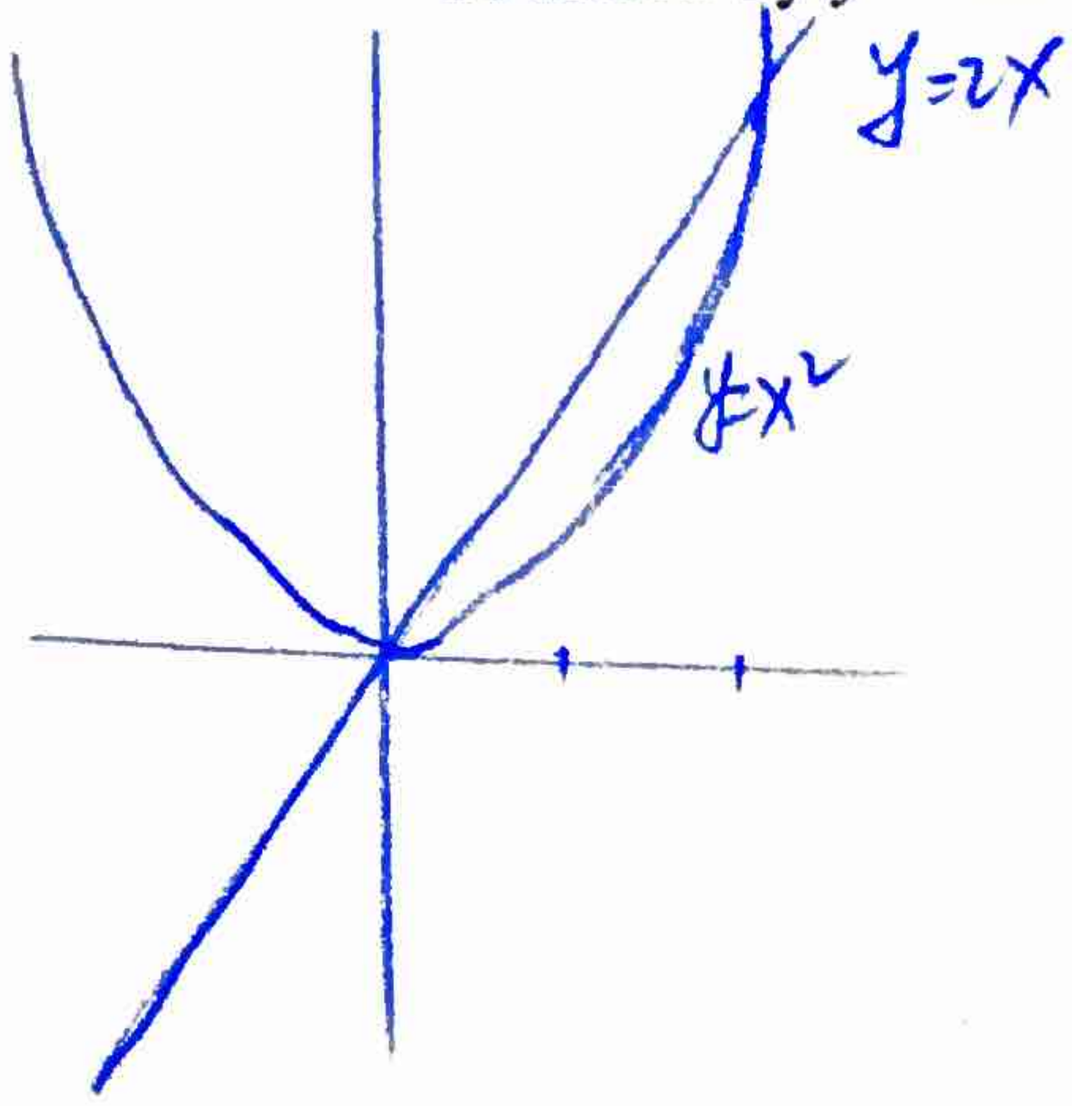
$u(1) = 82$

$u(3) = 406$

$$= \frac{1}{162} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_{82}^{406}$$

$$= \frac{1}{243}\left(406^{3/2} - 82^{3/2}\right)$$

2. (10 points) Calculate the center of mass (centroid) of a lamina with density $\rho = 6$ and shape R bounded by $y = 2x$ and the parabola $y = x^2$.



① The area of the region R is

$$A = \int_0^2 2x - x^2\, dx = x^2 - \frac{x^3}{3}\Big|_0^2 = \frac{4}{3}$$

The intersection,

$2x = x^2$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x=0$ and $x=2$

② $$\bar{x} = \frac{1}{A}\int_0^2 x(f(x) - g(x))\, dx$$

$$= \frac{3}{4}\int_0^2 x(2x - x^2)\, dx$$

$$= \frac{3}{4}\int_0^2 (2x^2 - x^3)\, dx$$

$$= \frac{3}{4}\left(\frac{2}{3}x^3 - \frac{x^4}{4}\right)\Big|_0^2 = \frac{3}{4}\left(\frac{2 \cdot 2^3}{3} - \frac{2^4}{4}\right) = 1$$

③ $$\bar{y} = \frac{1}{2A}\int_0^2 f(x)^2 - g(x)^2\, dx$$

$$= \frac{1}{2}\cdot\frac{3}{4}\int_0^2 4x^2 - x^4\, dx$$

$$= \frac{3}{8}\left(\frac{4}{3}x^3 - \frac{x^5}{5}\right)\Big|_0^2 = \frac{8}{5}$$

So, the centroid of the lamina is $(1, \frac{8}{5})$

3. (1) (6 points) For what values of r does the function $y = e^{rx}$ a solution for $y'' - 4y = 0$?

$y' = re^{rx}$

$y'' = r^2 e^{rx}$

plug into the equation

$r^2 e^{rx} - 4e^{rx} = 0$

$(r^2 - 4)e^{rx} = 0$

since $e^{rx} \neq 0$, we have

$r^2 - 4 = 0$

$(r+2)(r-2) = 0$

$r = 2$ and $r = -2$

So $y = e^{2x}$ and $y = e^{-2x}$ are solutions for $y'' - 4y = 0$

(2) (4 points) If r_1 and r_2 are the values of r that you found in part (1), show that every member of the family of functions $y = ae^{r_1 x} + be^{r_2 x}$ is a solution for $y'' - 4y = 0$.

$y = ae^{2x} + be^{-2x}$

$y' = 2ae^{2x} - 2be^{-2x}$

$y'' = 4ae^{2x} + 4be^{-2x}$

left side of the equation $= y'' - 4y$

$= 4ae^{2x} + 4be^{-2x} - 4(ae^{2x} + be^{-2x})$

$= 4ae^{2x} + 4be^{-2x} - 4ae^{2x} - 4be^{-2x}$

$= 0$

4. (10 points) Use Euler's method with step size $h = 0.2$ to solve $\frac{dy}{dx} = 2xy - 1$ with initial value $y(1) = 1$. Find $y(1.6)$.

$x_0 = 1$

$x_1 = x_0 + h = 1.2$

$x_2 = x_1 + h = 1.4$

$x_3 = x_2 + h = 1.6$

$y_0 = 1$

$y_1 = y_0 + h(2x_0 y_0 - 1)$

$= 1 + 0.2(2-1) = 1.2$

$y_2 = y_1 + h(2x_1 y_1 - 1)$

$= 1.2 + 0.2(2(1.2)(1.2) - 1)$

$= 1.576$

$y_3 = y_2 + h(2x_2 y_2 - 1)$

$= 1.576 + 0.2(2(1.4)(1.576) - 1)$

$= 2.25856$

So, $y(1.6) \approx y_3 = 2.25856$

5. (10 points) Solve the differential equation $\frac{dy}{dx} = 4y\cos x$ with the initial condition $y(0) = 3$.

$$\frac{dy}{y} = 4\cos x\, dx$$

$$\int \frac{1}{y}\, dy = 4\int \cos x\, dx$$

$$\ln|y| = 4\sin x + C$$

$$|y| = e^{4\sin x + C} = e^{\sin x} \cdot e^{C}$$

$$y = \pm e^{4\sin x} \cdot e^{C}$$

Denote $a = \pm e^{C}$, $\quad y = a \cdot e^{4\sin x}$

By initial condition $y(0) = 3$

$$3 = a \cdot e^{4\sin 0} = a$$

So, the solution is $y = 3e^{4\sin x}$

6. (2 bonus points) Solve the differential equation $(\sec^2 y)x^{-2}y' = \sin(2x^3)$

$$\sec^2 y\, dy = x^2 \sin 2x^3\, dx$$

$$\int \sec^2 y\, dy = \int x^2 \sin 2x^3\, dx$$

$$\tan y = -\frac{1}{6}\cos 2x^3 + C$$