University of Nevada, Reno —— MATH182 (Calculus 2)
Midterm 2 Test (50 points) — Spring 2019 —Version 2

Instructor: He Wang **Name:** Solutions

To receive full credit for a problem you must show **all necessary work** including which test used.

1. (6 points) Determine whether the **sequence** converges or diverges. If it converges, find the limit.

$$\left\{\frac{7n^2-40\sqrt{n}}{4n^2+30\sqrt{n}}\right\}$$

$$\lim_{n\to\infty}\frac{7n^2-40n^{\frac{1}{2}}}{4n^2+30n^{\frac{1}{2}}}=\lim_{n\to\infty}\frac{7-40\frac{1}{n^{3/2}}}{4+30\frac{1}{n^{3/2}}}=\frac{7}{4}$$

The sequence converges with limit $\frac{7}{4}$

2. (7 points) Determine whether the **series** converges or diverges. If it converges, find the sum.

$$\sum_{n=1}^{\infty}(-2)^n(5)^{1-n}$$

$$a_n=(-2)^n 5^{1-n}=\frac{(-2)^n}{5^{n-1}}=(-2)\left(-\frac{2}{5}\right)^{n-1}$$

So $\sum_{n=1}^{\infty}a_n$ is a geometric series with $a=-2$ and $r=-\frac{2}{5}$

$|r|=\frac{2}{5}<1$. So $\sum_{n=1}^{\infty}a_n$ is convergent.

The sum is $\frac{a}{1-r}=\frac{-2}{1-(-\frac{2}{5})}=-\frac{10}{7}$

V2

3. (6 points) Determine whether the series $\sum_{n=1}^{\infty} n^3(3/4)^n$ converges or diverges.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^3 (3/4)^{n+1}}{n^3 (3/4)^n} = \left(1+\frac{1}{n}\right)^3 \frac{3}{4} \to \frac{3}{4} < 1 \quad \text{when } n \to \infty.$$

By ratio test, $\sum_{n=1}^{\infty} n^3 (3/4)^n$ is absolutely convergent,

hence convergent.

4. (7 points) Find the **radius of convergence** and **interval of convergence** of the series $\sum_{n=0}^{\infty} \frac{(-6)^n x^n}{\sqrt[3]{n}}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{6^{n+1}|x|^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{6^n |x|^n} = 6\sqrt[3]{\frac{n}{n+1}}\,|x| = 6|x|\sqrt[3]{\frac{1}{1+\frac{1}{n}}}$$

So, $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 6|x|$

By ratio test, the series is absolutely convergent when $6|x|<1$.

$|x|<\frac{1}{6}$

· When $x=\frac{1}{6}$, the series $\sum_{n=0}^{\infty} \frac{(-6)^n (\frac{1}{6})^n}{\sqrt[3]{n}} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{n^{\frac{1}{3}}}$ is convergent.

When $x=-\frac{1}{6}$, the series $\sum_{n=0}^{\infty} \frac{(-6)^n(-\frac{1}{6})^n}{\sqrt[3]{n}} = \sum_{n=0}^{\infty} \frac{1}{n^{\frac{1}{3}}}$ is divergent.

So, the radius of convergence is $R=\frac{1}{6}$

the interval of convergence is $-\frac{1}{6} < x \leq \frac{1}{6}$

5. (6 points) Find a power series representation for the function $f(x) = \frac{x}{5-x^2}$ and determine the interval of convergence.

By geometric series $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad |x|<1$

$$\frac{1}{5-x^2} = \frac{1}{5}\cdot\frac{1}{1-\frac{x^2}{5}} = \frac{1}{5}\sum_{n=0}^{\infty}\left(\frac{x^2}{5}\right)^n \qquad \left|\frac{x^2}{5}\right|<1 \qquad |x|<\sqrt{5}$$

$$f(x) = \frac{x}{5-x^2} = \frac{x}{5}\sum_{n=0}^{\infty}\left(\frac{x^2}{5}\right)^n = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{5^{n+1}} \qquad |x|<\sqrt{5}$$

The interval of convergence is $-\sqrt{5}<x<\sqrt{5}$

6. (6 points) Use the **integral test** to determine whether the series $\sum_{n=1}^{\infty}\frac{\sqrt{\ln n}}{n}$ is convergent or divergent.

① $f(x) = \frac{\sqrt{\ln x}}{x} > 0$ when $x \geq 1$

② $f(x)$ is decreasing. $f'(x) = \frac{\sqrt{\ln x}\left(\frac{1}{2} - \ln x\right)}{x^2} < 0$ when $x>e$

③ $$\int_1^{\infty}\frac{\sqrt{\ln x}}{x}\,dx = \lim_{t\to\infty}\frac{2}{3}(\ln x)^{3/2}\Big|_1^t = \lim_{t\to\infty}\frac{2}{3}(\ln t)^{\frac{3}{2}} = \infty$$

divergent.

Let $u=\ln x$

$du = \frac{1}{x}dx$

$$\int\frac{\sqrt{\ln x}}{x}dx = \int u^{\frac{1}{2}}du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}(\ln x)^{\frac{3}{2}} + C$$

So, by integral test, $\sum_{n=1}^{\infty}\frac{\sqrt{\ln n}}{n}$ is divergent.

7. (2 points each, no partial points) Multi-choice. Determine whether the series is absolutely convergent, conditionally convergent or divergent.

(1.) $\sum_{n=1}^{\infty}(-1)^n\frac{3n^2-1}{5n^2+6}$ (1.) Answer: C

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

(2.) $\sum_{n=1}^{\infty}(-1)^n\frac{2}{\sqrt{3n-1}}$ (2.) Answer: B

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

(3.) $\sum_{n=1}^{\infty}\frac{5^n-3}{3^{2n}+5}$ (3.) Answer: A

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

(4.) $\sum_{n=1}^{\infty}\frac{\sin(n^8)}{n^3}$ (4.) Answer: A

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

(5.) $\sum_{n=2}^{\infty}\frac{1}{n\sqrt{\ln n}}$ (5.) Answer: C

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

(6.) $\sum_{n=1}^{\infty}(-1)^n\frac{n+1}{\sqrt{n^3+1}}$ (6.) Answer: B

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.