University of Nevada, Reno — MATH182 (Calculus 2)
Midterm 2 Test (50 points) — Spring 2019 —Version 1

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To receive full credit for a problem you must show **all necessary work** including which test used.

1. (6 points) Determine whether the **sequence** converges or diverges. If it converges, find the limit.

$$\left\{\frac{8n^2 - 30\sqrt{n}}{3n^2 + 50\sqrt{n}}\right\}$$

$$\lim_{n\to\infty} \frac{8n^2 - 30n^{\frac{1}{2}}}{3n^2 + 50n^{\frac{1}{2}}} = \lim_{n\to\infty} \frac{8 - 30\frac{1}{n^{3/2}}}{3 + 50\frac{1}{n^{3/2}}} = \frac{8}{3}$$

The sequence converges with limit $\frac{8}{3}$

2. (7 points) Determine whether the **series** converges or diverges. If it converges, find the sum.

$$\sum_{n=1}^{\infty} (-3)^n (5)^{1-n}$$

$$a_n = (-3)^n 5^{1-n} = \frac{(-3)^n}{5^{n-1}} = (-3)\left(-\frac{3}{5}\right)^{n-1}$$

So $\sum_{n=1}^{\infty} a_n$ is a geometric series with $a = -3$ and $r = -\frac{3}{5}$

$|r| = \frac{3}{5} < 1$ So, $\sum_{n=1}^{\infty} a_n$ is convergent.

The sum is $\frac{a}{1-r} = \frac{-3}{1-(-\frac{3}{5})} = -\frac{15}{8}$

V1

3. (6 points) Determine whether the series $\sum_{n=1}^{\infty} n^3(3/5)^n$ converges or diverges.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)^3\,(3/5)^{n+1}}{n^3(3/5)^n} = \left(1+\frac{1}{n}\right)^3 \cdot \frac{3}{5} \longrightarrow \frac{3}{5} < 1, \text{ when } n \to \infty.$$

By ratio test, $\sum_{n=1}^{\infty} n^3(3/5)^n$ is absolutely convergent, hence convergent.

4. (7 points) Find the **radius of convergence** and **interval of convergence** of the series $\sum_{n=0}^{\infty} \frac{(-5)^n x^n}{\sqrt[3]{n}}$.

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{5^{n+1}|x|^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{5^n|x|^n} = 5\sqrt[3]{\frac{n}{n+1}}\,|x| = 5\sqrt[3]{\frac{1}{1+\frac{1}{n}}}\,|x|$$

$$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 5|x|$$

So, by ratio test, the series is absolutely convergent when $5|x|<1$.

$|x| < \frac{1}{5}$

when $x = \frac{1}{5}$, the series $\sum_{n=0}^{\infty} \frac{(-5)^n(\frac{1}{5})^n}{\sqrt[3]{n}} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{\sqrt[3]{n}}$ is convergent.

when $x = -\frac{1}{5}$, the series $\sum_{n=0}^{\infty} \frac{(-5)^n(-\frac{1}{5})^n}{\sqrt[3]{n}} = \sum_{n=0}^{\infty} \frac{1}{\sqrt[3]{n}}$ is divergent.

So, the radius of convergence $R = \frac{1}{5}$

the interval of convergence is $-\frac{1}{5} < x \leq \frac{1}{5}$

V1

5. (6 points) Find a power series representation for the function $f(x) = \frac{x}{3-x^2}$ and determine the interval of convergence.

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad |x|<1 \quad \text{by geometric series}$$

$$\frac{1}{3-x^2} = \frac{1}{3} \cdot \frac{1}{1-\frac{x^2}{3}} = \frac{1}{3} \sum_{n=0}^{\infty} \left(\frac{x^2}{3}\right)^n \quad \left|\frac{x^2}{3}\right|<1 \quad |x|<\sqrt{3}$$

$$f(x) = \frac{x}{3-x^2} = \frac{x}{3} \sum_{n=0}^{\infty} \left(\frac{x^2}{3}\right)^n = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{3^{n+1}} \quad |x|<\sqrt{3}.$$

The interval of convergence is $|x|<\sqrt{3}$ or $-\sqrt{3}<x<\sqrt{3}$

6. (6 points) Use the **integral test** to determine whether the series $\sum_{n=1}^{\infty} \frac{\sqrt{\ln n}}{n}$ is convergent or divergent.

① $f(x) = \frac{\sqrt{\ln x}}{x} > 0$ when $x \geq 1$

② $f(x)$ is decreasing. $f'(x) = \frac{\sqrt{\ln x}\left(\frac{1}{2} - \ln x\right)}{x^2} < 0$ when $x > e$..

③ $\int_1^{\infty} \frac{\sqrt{\ln x}}{x}\, dx = \lim_{t\to\infty} \frac{2}{3}(\ln x)^{\frac{3}{2}}\Big|_1^t = \lim_{t\to\infty} \frac{2}{3}(\ln t)^{\frac{3}{2}} = \infty$ divergent.

$$\boxed{\begin{array}{l} \text{Let } u=\ln x \quad \int \frac{\sqrt{\ln x}}{x}\, dx = \int u^{\frac{1}{2}} du = \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}(\ln x)^{\frac{3}{2}} + C \\ du = \frac{1}{x} dx \end{array}}$$

So, by integral test, $\sum_{n=1}^{\infty} \frac{\sqrt{\ln n}}{n}$ is divergent.

V1

7. (2 points each, no partial points) Multi-choice. Determine whether the series is absolutely convergent, conditionally convergent or divergent.

A (1.) $\sum_{n=1}^{\infty} \frac{\sin(n^8)}{n^3}$

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

C (2.) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

B (3.) $\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{\sqrt{n^3+1}}$

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

C (4.) $\sum_{n=1}^{\infty} (-1)^n \frac{3n^2-1}{5n^2+6}$

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

B (5.) $\sum_{n=1}^{\infty} (-1)^n \frac{2}{\sqrt{3n-1}}$

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.

A (6.) $\sum_{n=1}^{\infty} \frac{5^n-3}{3^{2n}+5}$

(A) absolutely convergent. (B) conditionally convergent. (C) divergent.