University of Nevada, Reno — MATH182 (Calculus 2)
Midterm 1 Test (50 points) — Spring 2019 —Version 1

Instructor: He Wang **Name:** Solutions

To receive full credit for a problem you must show all necessary work.

1. (8 points) Find the derivative of the function: $f(x) = \int_1^{x^3-1} \cos(2t)\, dt$.

Suppose $F'(t) = \cos 2t$

So $f(x) = F(x^3-1) - F(1)$ by fundamental theorem.

So $f'(x) = F'(x^3-1) \cdot 3x^2 - 0$

$= \cos 2(x^3-1) \cdot 3x^2$

2. (8 points) Compute the indefinite integral $\int \frac{e^x}{\sqrt{e^x-2}}\, dx$

Let $u(x) = e^x - 2$

$du = e^x dx$

$dx = \frac{1}{e^x} du$

$$\int \frac{e^x}{\sqrt{e^x-2}} dx = \int \frac{e^x}{\sqrt{u}} \frac{1}{e^x} du$$

$$= \int u^{-\frac{1}{2}} du$$

$$= \frac{u^{\frac{1}{2}}}{1/2} + C$$

$$= 2(e^x-2)^{\frac{1}{2}} + C$$

or $= 2\sqrt{e^x-2} + C$

3. (8 points) Find $\int_e^{\infty} \frac{1}{x(\ln x)^2}\, dx$.

Step 1. Let $u = \ln x$

$du = \frac{1}{x} dx$

$dx = x\, du$

Step 2.

$\int \frac{1}{x(\ln x)^2} dx$

$= \int \frac{1}{x \cdot u^2} \cdot x\, du$

$= \int u^{-2} du$

$= -u^{-1} + C = -\frac{1}{\ln x} + c$

Step 3. $\int_e^{\infty} \frac{1}{x(\ln x)^2} dx$

$= \lim_{t \to \infty} -\frac{1}{\ln x}\Big|_e^t$

$= \lim_{t \to \infty} \left(-\frac{1}{\ln t}\right) - \left(-\frac{1}{\ln e}\right)$

$= 0 + 1$

$= 1$

4. (7 points) Find $\lim_{n\to\infty} \cos\left(\frac{n}{\sqrt{3n^3+2}}\right)$.

$\lim_{n\to\infty} \cos\left(\frac{n}{\sqrt{3n^3+2}}\right)$

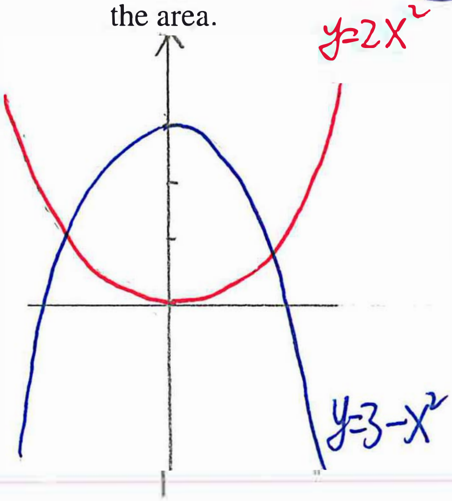
$= \lim_{n\to\infty} \cos\left(\frac{1}{\sqrt{3n+\frac{2}{n^2}}}\right)$

$= \cos\left(\lim_{n\to\infty} \frac{1}{\sqrt{3n+\frac{2}{n^2}}}\right)$

$= \cos 0$

$= 1$

5. (7 points) Sketch the region enclosed by the parabola $y = 2x^2$ and the line $y = 3 - x^2$, and find the area.



intersection points

$2x^2 = 3 - x^2$

$3x^2 = 3$

$x^2 = 1 \qquad x = -1 \text{ or } 1$

$$\text{Area} = \int_{-1}^{1} 3 - x^2 - 2x^2 \, dx$$

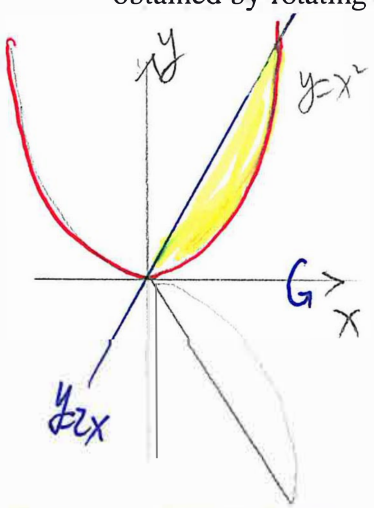
$$= \int_{-1}^{1} 3 - 3x^2 \, dx$$

$$= 3x - x^3 \Big|_{-1}^{1}$$

$$= (3-1) - (-3 - (-1))$$

$$= 4$$

6. (7 points). Let R be the region bounded by $y = x^2$ and $y = 2x$. Find the volume of the solid obtained by rotating R about the x-axis.



intersection points

$x^2 = 2x$

$x(x-2) = 0$

$x = 0 \quad x = 2$

$$A(x) = \pi (2x)^2 - \pi (x^2)^2$$

$$= \pi (4x^2 - x^4)$$

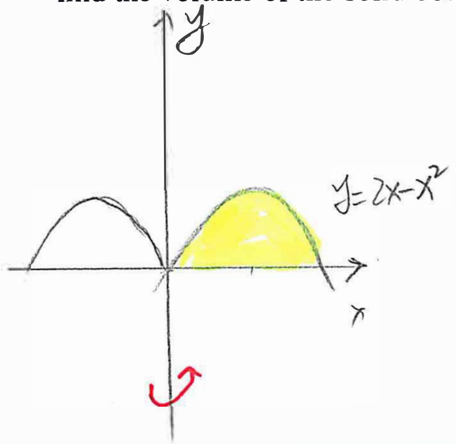
$$\text{Volume} = \int_0^2 A(x) \, dx$$

$$= \pi \int_0^2 4x^2 - x^4 \, dx$$

$$= \pi \left(\frac{4x^3}{3} - \frac{x^5}{5} \right) \Big|_0^2$$

$$= \pi \left(\frac{32}{3} - \frac{32}{5} \right) - 0 = \frac{64\pi}{15}$$

7. (7 points) The region R is enclosed by the curves $y = 2x - x^2$ and $y = 0$. Use cylindrical shells, find the volume of the solid obtained by rotating R about the y-axis.



intersection points $2x - x^2 = 0$

$x(2-x) = 0$

$x = 0$ or 2.

$$\text{Volume} = \int_0^2 2\pi x (2x - x^2)\, dx$$

$$= 2\pi \int_0^2 2x^2 - x^3\, dx$$

$$= 2\pi \left(\frac{2x^3}{3} - \frac{x^4}{4} \right) \Big|_0^2$$

$$= 2\pi \left(\frac{16}{3} - \frac{16}{4} \right)$$

$$= \frac{8\pi}{3}$$