Northeastern University, Department of Mathematics

MATH 5110: Applied Linear Algebra and Matrix Analysis. (Fall 2020, CRN 16759)

Instructor: He Wang

Email: he.wang@northeastern.edu

Course Webpage: on **Canvas**.

Class time and location 5:50 pm - 7:20 pm MW at Ryder Hall 265.

Office Hours: (on Zoom) Tuesday 8-9am, 5-7pm and other time by appointment..

Zoom Link for **office hours**: Join Zoom Meeting

https://northeastern.zoom.us/j/9798680433?pwd=SHpOOFFhYTJVVTRzemlVYnJNMTJBdz09

Meeting ID: 979 868 0433

Passcode: 2020HeWang

Recommended Texts and other Sources:

- *Finite-dimensional linear algebra*, Mark S. Gockenbach, CRC Press. (Primary source.)
- *Introduction to Linear Algebra*, Gilbert Strang, Wellesley-Cambridge Press
- *Applied Linear Algebra and Matrix Analysis,* Shores, Thomas S., Springer
- *Applied Linear Algebra,* Olver, Peter J., Shakiban, Chehrzad, Springer
- *A Second Course in Linear Algebra,* S. R. Garcia, R. A. Horn, Cambridge University Press
- *Matrix Analysis and Applied Linear Algebra*, C. D. Meyer, SIAM, 2000.
- Supplementary *lecture notes* will be available on the course webpage.

We use Gockenbach's book as the primary reference, however, text books are not required. Most of the Ebooks can be downloaded for free from Northeastern Library website. You can buy any (soft cover/student copy) printed book published by Springer from the link in Northeastern Library within $25 if the library bought the ebook.

This syllabus is subject to reasonable changes at the discretion of the instructor. (Check Canvas for the updated version.)

Recommended background: Undergraduate-level linear algebra. E.g., Math 2331 with textbook *Linear algebra with applications*, O. Bretscher. Some basic knowledge of understanding and writing proofs.

Overview: This course provides a rigorous treatment of the concepts and computational techniques of linear algebra. The major goal is to master the fundamental theory and techniques that underlie the applications of linear algebra in computer science, statistics, engineering, etc. Another goal is to provide a solid foundation for studying other subjects later. The course is designed for graduate students in Applied Mathematics, Computer Science, and students who wish to obtain deeper insights beyond an undergraduate level linear algebra.

List of main topics:

1. **Linear system, matrix algebra:** Linear systems and matrix equations; row reduced echelon form; existence and uniqueness of solutions; matrix operations including sum, scalar product, matrix product, inverse, transpose.

2. **Linear spaces over a field:** linear vector spaces; subspaces; spanning sets, independence, nullspace of matrix; basis and dimension; coordinate; rank-nullity theorem; linear transformations; change of basis; chain complex and homology.

3. **Jordan normal forms:** determinant; eigenvalues and eigenvectors; eigenbasis; eigenspaces; algebraic and geometric multiplicity; solution of iterated linear system; singularity, invertibility, rank etc; similarity; diagonalization; powers of a matrix; Jordan normal form and the Cayley Hamilton Theorem.

   Applications to Discrete dynamical systems: positive systems; Perron-Frobenius Theorem; stochastic matrix; Markov chain.

4. **Inner product spaces:** Inner product, norm, Cauchy-Schwarz inequality; orthogonal projection onto subspace; orthogonal basis; Gram-Schmidt process; symmetric matrix, diagonalization; positive definite; variational formulas for eigenvalues, Singular value decomposition and data compression.

   Applications to principal component analysis (PCA) for data analysis mean, optimal orthogonal projections, principal components, PCA in Statistics, covariance matrix, optimal linear combinations of random variables, principal components, interpretation.

5. **Extra topics** (as time allows): Matrix factorization, including Jordan, Schur, LU, QR, SVD; Calculus for matrix-valued functions; Neural networks and back propagation equation; etc.

Software: MATLAB will be used throughout the course. Students should be prepared to use *Matlab* for assignments and for projects. The first project will be (mostly) a Matlab tutorial.

Northeastern University has an active MATLAB and Simulink Campus-Wide License. Go to https://www.mathworks.com/products/matlab.html and use your Northeastern email address to create an account. You should be able get a free license for the Matlab product. Download and install Matlab on your computer.

Grade breakdown:

1. Homework (20%): There will be 6 homework assignments which will focus on the theory. Homework will be assigned each week or two. You have one week to finish the homework. In addition to being mathematically correct, your write-ups are expected to show a high level of clarity and completeness.

2. Computer Labs/Projects (25%): There will be roughly 5 labs will focus on the applications to real world problems using Matlab. Computer labs are designed to help you learn the concepts and techniques of linear algebra by using them in an computer environment and master the computational tools for important applications of linear algebra. Labs will be graded based on completeness, correctness and clarity in written.

3. Tests/Quizzes (25%): There will be 3 tests in class.

4. Final exam (30%). There is a comprehensive final exam in the end of the semester.

To receive full credit, homework/lab assignments must normally be handed in when due. Assignments may be turned in late with a valid excuse; this should be discussed with me and approved in advance if possible or otherwise at the earliest opportunity.

Exam/Tests makeup policy: Makeup exams will be allowed only in the event of a documented medical or other unforeseen emergency. You are responsible for avoiding foreseeable conflicts with the exams.

Collaboration: You are welcome, even encouraged, to collaborate on the homework assignments, but you are expected to write answers yourself and understand everything that you hand in. Collaboration is not allowed on the quizzes and exams.

Classroom Recording: This course, or parts of this course, may be recorded by instructor for educational purposes. These recordings will be made available on Canvas only to students enrolled in the course, the TAs and instructors, and other math department or administrative personnel for training, oversight, or evaluation purposes. If you have any concerns, please let me know.

Only students who have arranged an accommodation with the Disability Resource Center may use mechanical or electronic transcribing, recording, or communication devices in the classroom. Students with disabilities who believe they may need such an accommodation may contact the Disabilities Resource Center.

University Academic Integrity Policy: The university's academic integrity policy at OSCCR (http://www.northeastern.edu/osccr/academic-integrity-policy) discusses actions regarded as violations and their consequences for students.

Title IX: The University strictly prohibits sex or gender discrimination in all university programs and activities. Information on how to report an incident of such discrimination (which includes sexual harassment and sexual assault) is located at http://www.northeastern.edu/titleix.

Students with Disabilities: Students who have disabilities who wish to receive academic services and accommodations should follow the standard Disabilities Resource Center (DRC) procedures, http://www.northeastern.edu/drc/getting-started-with-the-drc.

College of Science Policies: The current College of Science Academic Course Policies are available at

https://cos.northeastern.edu/wp-content/uploads/2012/10/COS-teaching-policies-April-2017.pdf

TRACE: Every student is expected to complete the online TRACE survey at the end of the semester.

A tentative schedule of topics and tasks:

(Will be updated as course proceeds. Check Canvas for updated schedule)

Gockenbach's book is only a guide line. Several books in the recommended texts are very good sources. Each book has its advantages and drawbacks. I will provide my lecture notes for this class.

Week 1: (Gockenbach §1) (one day)

Linear system, from set to field, Gaussian elimination.

Computer Lab 0

Week 2: (Gockenbach §2 & part of §3)

Matrix algebra.

Assignment 1

Computer Lab 1

Week 3: (Gockenbach §3)

Vector spaces over fields, subspaces.

Linear transformation and their matrices,

Computer Lab

Week 4: (Gockenbach §3)

Independence, basis and dimension.

change of coordinates

Assignment 2

Week 5: (Gockenbach §4)

Determinants. Diagonalization.

Test 1

Week 6: (one day)

Oct 12: Columbus Day/Indigenous People's Day, no classes

Eigenvalues and eigenspaces.

Week 7: (Gockenbach §5)

Jordan canonical forms.

Computer Lab

Assignment 3

Week 8:

Discrete Dynamical System; Perron-Frobenius Theorem; Markov chain

Assignment 4

Test 2

Week 9: (Gockenbach §6)

Inner Products, Projections, Orthogonal basis, Gram-Schmidt process, QR factorization,

Computer Lab

Week 10: (Gockenbach §7) (one day)

Nov 11: Veterans Day, no classes

Least-squares approximation.

Assignment 5

Week 11: (Gockenbach §8)

The spectral theory of symmetric matrices,

Week 12: (one day)

Nov 25: First Day of Thanksgiving recess, no classes

Quadratic forms, positive and positive-definite transformations and matrices.

Computer Lab

Assignment 6

Weeks 13

The singular value decomposition; Principal component analysis

Test 3

Weeks 14 Review and extra topics

Computer lab topics:

Solving Linear Equations

Recover matrix from data

Change of coordinates for circular orbit

Area and determinant

Discrete Dynamical Systems

Least Squares Data Fitting

Digital Image Compression (SVD)

etc.