Solution

MATH 1231 Quiz 9 (A) (50pts)
Fall 2014

Name: ____________________

1. Let $E(t) = \frac{4}{x} + \frac{1}{\sqrt{x}}$ $= \frac{4}{x} + x^{-\frac{1}{2}}$ be the **rate of change** of Orange corporation's profit in million of dollars per year, t years after the year 2000.

(a) (6 points) Find the value of $\int_3^7 E(t)\, dt$ by the fundamental theorem in calculus. (Do not use fnInt.) Show all work with unit.

$$\int_3^7 E(t)\, dt = 4\ln|x| + 2x^{\frac{1}{2}} \Big|_3^7 = 5.217 \text{ million of dollars}$$

(b) (4 points) Write a sentence explaining the practical meaning of the integral $\int_3^7 E(t)\, dt$.

From 2003 to 2007, the Orange corporation's profit increase 5.217 millions \$.

2. Since 2000, the sales of Pear corporation have been changing at the rates (thousands of dollars per year) given in the following table.

Years since 2000	0	2	4	6	8	10
Sales rate (thousands of \$ per year)	8	45	76	83	73	42

(a) (4 points) Use the table above to find a quadratic model for $W(x)$, the rate of change of sales x years after 2000. Give your model with three decimal places.

$W(x) = y = ax^2 + bx + c$ thousands of dollars per year

$a = -2.25$
$b = 26.229$
$c = 5.857$

(b) (6 points) Using the model in part (a), write down a definite integral which gives the change in sales of Pear corporation between 2001 and 2007. Compute this definite integral.

$$\int_1^7 W(x)\, dx \approx 408.129 \text{ thousend of \$}$$

or 408.138

3. (10 points) The amount of money in a savings account t years after January 1, 2000 is given by the formula $A(t) = 15000e^{0.03t}$ dollars. Use a definite integral to find the **average amount of money** in the account between January 1, 2000 and January 1, 2006, rounded to the nearest cent.

$$\frac{\int_0^6 A(t)\, dt}{6-0} = \frac{98608.6815}{6} = 16434.78 \text{ dollars}$$

4. (10 points) Use the method of u-substitution to evaluate the following integral:

$$\int 4x^2\sqrt{4x^3-7}\,dx.$$

$u = 4x^3 - 7$

$du = 12x^2 dx$

$dx = \frac{1}{12x^2} du$

$= \int 4x^2 \cdot u^{\frac{1}{2}} \frac{1}{12x^2} du$

$= \frac{1}{3} u^{\frac{3}{2}} \cdot \frac{2}{3} + C$

$= \frac{2}{9} u^{\frac{3}{2}} + C = \frac{2}{9}(4x^3-7)^{\frac{3}{2}} + C$

In the following questions, circle the correct answer. In each question there is only one correct answer and there is no partial credit.

5. (5 points) The general antiderivative of $f(x) = \frac{1}{x^2} + e^3 - \sqrt[3]{x^2}$ is

(i) $x^{-1} + e^3x - \frac{5}{3}x^{\frac{5}{3}} + C$ (ii) $-x^{-1} + e^3x - \frac{5}{3}x^{\frac{5}{3}} + C$ (iii) $-x^{-1} + e^3x - \frac{3}{5}x^{\frac{5}{3}} + C$

(iv) $-x^{-1} + \frac{e^x}{3} - \frac{3}{5}x^{\frac{5}{3}} + C$ (v) None of these

6. (5 points) The general antiderivative of $f(x) = 3.1^x + 3.9x^3 - e^{-x}$ is

(i) $\frac{3.1^x}{\ln 3.1} + 3.9x^4 + e^{-x} + C$ (ii) $3.1^x(\ln 3.1) + 0.975x^4 + e^{-x} + C$ (iii) $\frac{3.1^x}{\ln 3.1} + 0.975x^4 - e^{-x} + C$

(iv) $\frac{3.1^x}{\ln 3.1} + 0.975x^4 + e^{-x} + C$ (v) None of these

MATH 1231 Quiz 9 (B) (50pts)
Fall 2014

Name: ______________________

1. Let $E(t) = \frac{3}{x} + \frac{1}{\sqrt{x}}$ be the **rate of change** of Orange corporation's profit in million of dollars per year, t years after the year 2000.

(a) (6 points) Find the value of $\int_3^8 E(t)\,dt$ by the fundamental theorem in calculus. (Do not use fnInt.) Show all work with unit.

$\int_3^8 E(t)\,dt = 3\ln|x| + 2x^{\frac{1}{2}} \Big|_3^8 = 5.135$ million of dollars

(b) (4 points) Write a sentence explaining the practical meaning of the integral $\int_3^8 E(t)\,dt$.

From 2003 to 2008, the orange corporation's profit increase 5.135 million $

2. Since 2000, the sales of Pear corporation have been changing at the rates (thousands of dollars per year) given in the following table.

Years since 2000	0	2	4	6	8	10
Sales rate (thousands of $ per year)	8	45	76	83	73	42

(a) (4 points) Use the table above to find a quadratic model for $W(x)$, the rate of change of sales x years after 2000. Give your model with three decimal places.

$W(x) = ax^2 + bx + c$ thousands of dollars per year

$a = -2.25$
$b = 26.229$
$c = 5.857$

(b) (6 points) Using the model in part (a), write down a definite integral which gives the change in sales of Pear corporation between 2001 and 2007. Compute this definite integral.

$\int_1^7 W(x)\,dx \approx 408.129$ thousand of dollars

or 408.138

3. (10 points) The amount of money in a savings account t years after January 1, 2000 is given by the formula $A(t) = 15000e^{0.03t}$ dollars. Use a definite integral to find the **average amount of money** in the account between January 1, 2000 and January 1, 2006, rounded to the nearest cent.

$$\frac{\int_0^6 A(t)\,dt}{6-0} = \frac{98608.6815}{6} \approx 16434.78 \text{ dollars}$$

4. (10 points) Use the method of u-substitution to evaluate the following integral:

$$\int 4x^3\sqrt{3x^4-5}\,dx.$$

$u=3x^4-5$

$du=12x^3dx$

$dx=\frac{1}{12x^3}du$

$= \int 4x^3 \cdot u^{\frac{1}{2}} \frac{1}{12x^3}\,du$

$= \frac{1}{3}u^{\frac{3}{2}} \cdot \frac{2}{3} + C = \frac{2}{9}(3x^4-5)^{\frac{3}{2}} + C$

In the following questions, circle the correct answer. In each question there is only one correct answer and there is no partial credit.

5. (5 points) The general antiderivative of $f(x) = 3.1^x + 3.9x^3 - e^{-x}$ is

(i) $\frac{3.1^x}{\ln 3.1} + 3.9x^4 + e^{-x} + C$ (ii) $3.1^x(\ln 3.1) + 0.975x^4 + e^{-x} + C$ (iii) $\frac{3.1^x}{\ln 3.1} + 0.975x^4 - e^{-x} + C$

(iv) $\frac{3.1^x}{\ln 3.1} + 0.975x^4 + e^{-x} + C$ (v) None of these

6. (5 points) The general antiderivative of $f(x) = \frac{1}{x^2} + e^3 - \sqrt[3]{x^2}$ is

(i) $x^{-1} + e^3x - \frac{5}{3}x^{\frac{5}{3}} + C$ (ii) $-x^{-1} + e^3x - \frac{5}{3}x^{\frac{5}{3}} + C$ (iii) $-x^{-1} + e^3x - \frac{3}{5}x^{\frac{5}{3}} + C$

(iv) $-x^{-1} + \frac{e^x}{3} - \frac{3}{5}x^{\frac{5}{3}} + C$ (v) None of these