Solution

MATH 1231 Quiz 6 (50pts)
Fall 2014

Name: ______________________

This quiz requires the TI 83/84. Always tell which function is in Y_1, Y_2, etc. in your calculator. When you use `nDeriv` to solve a problem say so.

1. A florist is trying to price her tulips for the spring season. She sells them by the dozen. A marketing poll has yielded the following model for the number of dozen tulips she can expect to sell. Let x stand for the selling price in dollars of a dozen tulips. If $4 \leq x \leq 22$, the demand function $D(x)$ is given by the exponential function:

$$D(x) = 506.54(.908)^x$$

Enter this formula carefully into Y_1 on your calculator. Check that you have entered the formula correctly by computing the demand if $x = 13$. Your answer should be 144.4540681.

(a) (8 points) Each dozen tulips costs the florist \$3.75 and the florist has fixed costs of \$65.00 for advertising. Write down the formulas for $R(x)$, $C(x)$, and $P(x)$ (the revenue, cost and profit functions in dollars). **Write each formula out in full. Do not use any abbreviations.**

$$R(x) = x \cdot D(x) = x \cdot 506.54(0.908^x)$$

$$C(x) = 3.75D(x) + 65 = 3.75 \times 506.54(0.908^x) + 65$$

$$P(x) = R(x) - C(x) = (x - 3.75)\,506.54(0.908^x) - 65$$

(b) (8 points) Use `nDeriv` to determine the price that maximizes **REVENUE**. Write the answer with all the decimal places the calculator gives.

- **Calculator answer:** 10.361524
- Circle the equation you solved:

(i) $\texttt{nDeriv}(P(x), x, x) = 0$ (ii) $\texttt{nDeriv}(R(x), x, x) = R(x)$ (iii) $\texttt{nDeriv}(D(x), x, x) = 0$

(iv) $\texttt{nDeriv}(R(x), x, x) = 0$ (v) $\texttt{nDeriv}(R'(x), x, x) = 0$

- Circle the calculator procedure you used:

(i) `2nd/Calc/Zero` (ii) `2nd/Calc/Intersect` (iii) `SOLVER`

(c) (5 points) Use the **second** derivative test to show that the price you found in part (b) maximizes the revenue.

$Y1 = D(x)$

$Y2 = x \cdot Y1$

$Y5 = \frac{d}{dx}(Y2)\Big|_{x=x} = \text{nDeriv}(Y2, x, x)$

$Y6 = \frac{d}{dx}(Y5)\Big|_{x=x}$

$R''(10.361524) = Y6(10.361524)$

$= -17.98 < 0$

So $R(x)$ is Concave down

So $x = 10.361524$ is a relative maximum point by the 2ed derivative Test

$Y_1 = D(x)$
$Y_2 = X Y_1$
$Y_3 = 3.75 Y_1 + 65$
$Y_4 = Y_2 - Y_3$

(d) (5 points) Round the selling price from part (b) to the nearest penny and fill in the following table with the values when **REVENUE** is maximized. Note that demand must be a whole number.

	Selling Price	Demand	Revenue	Cost	Profit
Value:	10.36	186	1930.82	763.90	1166.93
Unit:	\$	dozen	\$	\$	\$

(e) (6 points) Use `nDeriv` to determine the price that maximizes **PROFIT**. Show your work **especially the equation you solve and tell how you use the calculator.** Write the answer with all the decimal places the calculator gives.

equation to solve: $P'(x) = 0$ or nDeriv(Y4, X, X) = 0

or nDeriv(P(x), X, X) = 0

calculator procedure: 2nd / calc / zero

calculator answer: 14.111524

(f) (5 points) Use the **first** derivative test to show that the answer in part (e) maximizes the profit.

x	14	15
$P'(x)$	1.4	−10.2

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So by the first derivative Test then 14.111524 is a maximum.

2. Consider the function $g(x) = x^3 - 7.5x^2 + 10x + 5$.

(a) (5 points) Find the **inflection point** of the function $g(x)$ by computing the second derivative **by hand.** Give both x and y coordinates.

$g'(x) = 3x^2 - 15x + 10$

$g''(x) = 6x - 15 = 0$

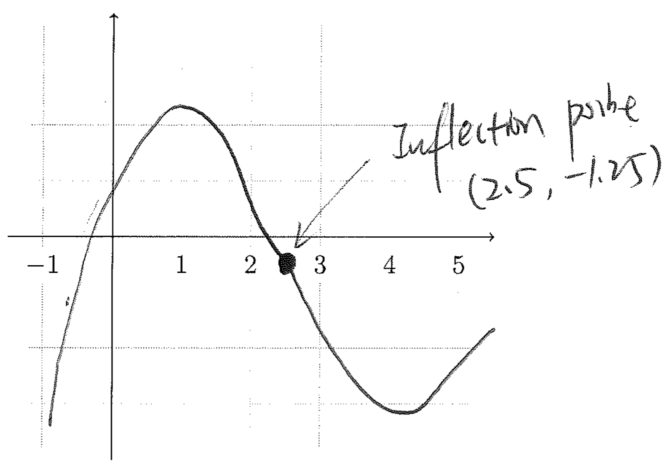
$x = \frac{15}{6} = \frac{5}{2} = 2.5$

$g(2.5) = -1.25$

The Inflection point is (2.5, −1.25

(b) (5 points) Sketch the graph of $g(x)$ over the interval $-1 \le x \le 5$. Label the inflection point of $g(x)$ in your sketch. Give the window you use.

$X_{min} =$ −1, $X_{max} =$ 5, $Y_{min} =$ −13.5, $Y_{max} =$ 8.712.



(c) (3 points) Use the graph to decide what kind of inflection point $g(x)$ has. Circle one of the following.

(A) Point of fastest increase

(B) Point of slowest increase

(C) Point of fastest decrease

(D) Point of slowest decrease