Solution

MATH 1231 Quiz 5 (50pts) A
Fall 2014

Name: ______________________

Calculators are NOT permitted.

1. The company that produces keychains for the Northeastern University Bookstore has revised their demand functions in light of new information. When the price is x dollars per keychain the demand for keychains is $D(x) = 10 - x$ **hundred** keychains.

(a) (5 points) If the unit cost of each keychain is 2 dollars, write the formula for the **profit**, with **units**.

$R(x) = x \cdot D(x) = x(10-x)$

$C(x) = 2 \cdot D(x) = 20 - 2x$

$P(x) = R(x) - C(x) = 10x - x^2 - (20-2x)$

$= -x^2 + 12x - 20$ hundred \$

(b) (3 points) Use the derivatives and algebra to find the price that maximizes profit.

$P'(x) = -2x + 12 = 0$ $\quad x = 6$ \$

(c) (5 points) Use the **first derivative** test **and** the **second derivative** test show that the price you found in (b) gives the maximum profit.

1st derivative test

x	5	7
$f'(x)$	2	-2

$+$ $-$

2ed derivative test

$p''(x) = -2$

$p''(6) = -2 < 0$ concave down

So $x=6$ is a relative maximum.

2. Find the **second** derivative of each function:(6 points each)

(a) $\quad g(x) = 2x^3 - \frac{2}{x} + 3\ln x + 5e^x = 2x^3 - 2x^{-1} + 3\ln x + 5e^x$

$g'(x) = 6x^2 + 2x^{-2} + \frac{3}{x} + 5e^x$

$g''(x) = 12x - 4x^{-3} - 3x^{-2} + 5e^x$

(b) $\quad h(x) = \ln(x^2 + 3)$

$h'(x) = \dfrac{2x}{x^2+3}$ (chain rule)

$= (2x)(x^2+3)^{-1}$

$h''(x) = 2x\left(-(x^2+3)^{-2} \cdot 2x\right) + 2(x^2+3)^{-1}$

$2x \quad 2$

$(x^2+3)^{-1} \quad -(x^2+3)^{-2} \cdot 2x$

(c) $h(x) = 3x \cdot e^x$

$3x \quad 3$

$e^x \quad e^x$

$h'(x) = 3xe^x + 3e^x$

$h''(x) = 3xe^x + 3e^x + 3e^x = (3x+6)e^x$

A Solution

3. The cubic function $j(x)$ has critical points at $x = 13$ and $x = 18$. Values of $j''(x)$, the **second** derivative of $j(x)$ are given in the table below.

x	10	13	15	18	22
$j''(x)$	10	4	0	-6	-14

(a) (5 points) Which (if any) of the critical points of $j(x)$ are relative **maxima** of $j(x)$? Explain your **reasoning**.

$x=18$ is a relative maximum, since $j''(18) = -6 < 0$. Concave down. $\therefore$ it follows by the second derivative Test.

(b) (5 points) Which (if any) of the critical points of $j(x)$ are relative **minima** of $j(x)$? Explain your **reasoning**.

$x=13$ is a relative minimum. since $j''(13) = 4 > 0$, concave up. the result $\therefore$ follows by the second derivative Test

4. Suppose that for a certain function $f(x)$, you are told the **derivative** is $f'(x) = x^3 - 2x^2$. (Circle the correct result, and show work)

(a) (5 points) The critical point $x = 0$ is a (A) relative max (B) relative min (C) neither

Work:

x	-1	1
$f'(x)$	-3	-1

There is No sign change at $x=0$ for $f'(x)$. So $\therefore$ the result follows by the first deirvative Test.

(b) (5 points) The critical point $x = 2$ is a (A) relative max (B) relative min (C) neither

Work:

x	1	3
$f'(x)$	-1	9

$-$ $+$ By the first derivative Test.

5. The function $f(x) = x^2 + \dfrac{b}{x}$, where b is a positive constant, and the domain of $f(x)$ is $x > 0$. Suppose that $f(x)$ has a critical point at $x = 1$. $f'(1)=0$

(a) (3 points) Find the value of b. Show work.

$f'(x) = 2x - bx^{-2}$

$f'(1) = 2 - b = 0$

$b = 2$.

(b) (2 points) The critical point $x = 1$ is a

(A) relative max (B) relative min (C) neither (Circle the correct result)

$f''(x) = 2 + 2bx^{-3}$
$= 2 + 4x^{-3}$

$f''(1) = 6 > 0$

B Solution

3. The cubic function $j(x)$ has critical points at $x = 12$ and $x = 17$. Values of $j''(x)$, the **second** derivative of $j(x)$ are given in the table below.

x	10	12	15	17	21
$j''(x)$	10	7	0	-5	-12

(a) (5 points) Which (if any) of the critical points of $j(x)$ are relative **minima** of $j(x)$? Explain your **reasoning**.

$x=12$ Since $j''(12)=7>0$, So $j(x)$ is Concave up at $x=12$ $\cup$.

So $x=12$ is a relative minimum by the 2ed derivative test

(b) (5 points) Which (if any) of the critical points of $j(x)$ are relative **maxima** of $j(x)$? Explain your **reasoning**.

$x=17$ Since $j''(17)=-5<0$. So $j(x)$ is concave down at $x=17$ $\cap$

So $x=17$ is a relative maximum by the 2ed derivative test

4. Suppose that for a certain function $f(x)$, you are told the **derivative** is $f'(x) = x^3 - 2x^2$. (Circle the correct result, and show work)

(a) (5 points) The critical point $x = 0$ is a (A) relative min (B) relative max (C) neither

Work:

x	-1	1
$f'(x)$	-3	-1

There is No sign change at $x=0$ for $f'(x)$

So the result follows by the first derivative test.

(b) (5 points) The critical point $x = 2$ is a (A) relative min (B) relative max (C) neither

Work:

x	1	3
$f'(x)$	-1	9

$-\cup+$

By the first derivative Test.

5. The function $f(x) = x^2 + \dfrac{b}{x}$, where b is a positive constant, and the domain of $f(x)$ is $x > 0$. Suppose that $f(x)$ has a critical point at $x = 1$.

(a) (3 points) Find the value of b. Show work.

$f'(x)=2x-bx^{-2}$

$f'(1)=2-b=0$

$b=2$

(b) (2 points) The critical point $x = 1$ is a

✓ (A) relative min ~~(B)~~ relative max (C) neither (Circle the correct result)

$f''(x)=2+2bx^{-3}$

$=2+4x^{-3}$

$f''(1)=6>0$ $\cup$

B Solution

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$C(x) = 2 \cdot D(x) = 20 - 2x$ $= -x^2 + 12x - 20$ hundred \$

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(c) (5 points) Use the **first derivative** test **and** the **second derivative** test show that the price you found in (b) gives the maximum profit.

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2. Find the **second** derivative of each function:(6 points each)

(a) $g(x) = 2x^3 - \frac{2}{x} + 4\ln x + 7e^x$ $\frac{2}{x} = 2\cdot x^{-1}$

$g'(x) = 6x^2 + 2x^{-2} + \frac{4}{x} + 7e^x$

$g''(x) = 12x - 4x^{-3} - 4x^{-2} + 7e^x$

(b) $h(x) = \ln(x^2 + 2)$

$h'(x) = \frac{2x}{x^2+2}$ chain rule

$= (2x)(x^2+2)^{-1}$

$h''(x) = 2x(-(x^2+2)^{-2} \cdot 2x) + 2(x^2+2)^{-1}$

(c) $h(x) = 2x\,e^x$

$2x \quad 2$
$e^x \times e^x$

$h'(x) = 2xe^x + 2e^x$

$h''(x) = 2xe^x + 2e^x + 2e^x$

$= (2x+4)e^x$