A Solution

MATH 1231 Quiz 4 (50pts)
Fall 2014

Name: _____

Find the derivative of each function 1-5. **Show all your work.** (**NOT** require simplification.) (8 points each)

1. $j(x) = 10\ln(5x^{-4} - 2x^3)$

$u(x) = 5x^{-4} - 2x^3 \qquad u'(x) = -20x^{-5} - 6x^2$

$f(u) = 10\ln(u) \qquad f'(u) = \frac{10}{u}$

$j'(x) = \frac{10}{5x^{-4} - 2x^3} \cdot (-20x^{-5} - 6x^2)$

2. $f(x) = (2x^6 - 5e^3)(3e^{2x} - 4(3^x))$

$g(x) = 2x^6 - 5e^3 \qquad g'(x) = 12x^5$

$h(x) = 3e^{2x} - 4(3^x) \qquad h'(x) = 6e^{2x} - 4(\ln 3)3^x$

$f'(x) = (2x^6 - 5e^3)(6e^{2x} - 4(\ln 3)3^x) + (3e^{2x} - 4(3^x))(12x^5)$

3. $f(x) = \frac{9.8}{1 + e^{-0.48x}} + x^2$

$u(x) = 1 + e^{-0.48x}$

$g(u) = \frac{9.8}{u} = 9.8u^{-1} \qquad f'(x) = -9.8(1 + e^{-0.48x})^{-2}(-0.48e^{-0.48x}) + 2x$

$u'(x) = -0.48e^{-0.48x}$

$g'(u) = -9.8u^{-2}$

4. $h(x) = \frac{2x^5 - 3^x}{2\sqrt[3]{x^4}}$.

$= (2x^5 - 3^x)\left(\frac{1}{2}x^{-\frac{4}{3}}\right)$

$f(x) = 2x^5 - 3^x \qquad f'(x) = 10x^4 - (\ln 3)3^x$

$g(x) = \frac{1}{2}x^{-\frac{4}{3}} \qquad g'(x) = -\frac{2}{3}x^{-\frac{7}{3}}$

$h'(x) = (2x^5 - 3^x)\left(-\frac{2}{3}x^{-\frac{7}{3}}\right) + \left(\frac{1}{2}x^{-\frac{4}{3}}\right)(10x^4 - (\ln 3)3^x)$

A Solution

5. $f(x) = (10\ln(5x^{-4} - 2x^3))(4e^x + 3x^5)$

$g(x) = 10\ln(5x^{-4} - 2x^3)$ $\quad g'(x) = \dfrac{10(-20x^{-5} - 6x^2)}{5x^{-4} - 2x^3}$

$h(x) = 4e^x + 3x^5$ $\quad h'(x) = 4e^x + 15x^4$

$$f'(x) = 10\ln(5x^{-4} - 2x^3)(4e^x + 15x^4) + (4e^x + 3x^5)\frac{10(-20x^{-5} - 6x^2)}{5x^{-4} - 2x^3}$$

6. (10 points) The number of Americans age 65 or older for the years 1995 through 2030 can be modeled by

$$N(x) = 0.03x^2 + 0.35x + 34.52 \qquad \text{million people,}$$

where x is the number of years after 2000.

(a) Write out the rate-of-change formula for the number of Americans age 65 or older.(With **unit**.)

$N'(x) = 0.06x + 0.35$ million people / year

(b) Fill in the following table. Round numerical approximations to **two** decimal places.

	1995	2011	2030	Units
x	-5	11	30	year
Number of Americans age 65 or older	33.52	~~37.52~~ 42	72.02	million people
Rate of change of number of Americans age 65 or older	0.05	~~0.71~~ 1.01	2.15	million people per year

B Solution

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Name: _____

Find the derivative of each function 1-5. **Show all your work.** (**NOT** require simplification.)
(8 points each)

1. $j(x) = 10\ln(5x^{-4} - 3x^2)$

~~$u'(x) = -20x^{-5}$~~

$u(x) = 5x^{-4} - 3x^2 \qquad u'(x) = -20x^{-5} - 6x \qquad j'(x) = \dfrac{10(-20x^{-5} - 6x)}{5x^{-4} - 3x^2}$

$f(u) = 10\ln(u) \qquad f'(u) = \dfrac{10}{u}$

2. $f(x) = (3x^6 - 5e^2)(3e^{2x} - 4(3^x))$

$g(x) = 3x^6 - 5e^2 \qquad g'(x) = 18x^5$

$h(x) = 3e^{2x} - 4(3^x) \qquad h'(x) = 6e^{2x} - 4(\ln 3)3^x$

$f'(x) = (3x^6 - 5e^2)(6e^{2x} - 4(\ln 3)3^x) + (3e^{2x} - 4(3^x))(18x^5)$

3. $h(x) = \dfrac{3x^4 - 5^x}{2\sqrt[3]{x^4}}$.

$= (3x^4 - 5^x)\left(\frac{1}{2}x^{-\frac{4}{3}}\right)$

$f(x) = 3x^4 - 5^x \qquad f'(x) = 12x^3 - (\ln 5)5^x$

$g(x) = \frac{1}{2}x^{-\frac{4}{3}} \qquad g'(x) = -\frac{2}{3}x^{-\frac{7}{3}}$

$h'(x) = (3x^4 - 5^x)\left(-\frac{2}{3}x^{-\frac{7}{3}}\right) + \left(\frac{1}{2}x^{-\frac{4}{3}}\right)\left(12x^3 - (\ln 5)5^x\right)$

4. $f(x) = \dfrac{9.7}{1 + e^{-0.47x}} + x^2$

$u(x) = 1 + e^{-0.47x}$

$g(u) = \dfrac{9.7}{u} = 9.7u^{-1}$

$u'(x) = .047e^{-0.47x}$

$g'(u) = -9.7u^{-2}$

$f'(x) = g'(u) \cdot u'(x)$

$= -9.7(1 + e^{-0.47x})^{-2}(-.47e^{-.47x})$

$+2x$

B Solution

5. $f(x) = (10\ln(5x^{-4} - 3x^2))(3e^x + 4x^5)$

$g(x) = 10\ln(5x^{-4} - 3x^2)$ $\quad g'(x) = \dfrac{10(-20x^{-5} - 6x)}{5x^{-4} - 3x^2}$

$h(x) = 3e^x + 4x^5$ $\quad h'(x) = 3e^x + 20x^4$

$$f'(x) = (10\ln(5x^{-4} - 3x^2))(3e^x + 20x^4) + (3e^x + 4x^5)\frac{10(-20x^{-5} - 6x)}{5x^{-4} - 3x^2}$$

6. (10 points) The number of Americans age 65 or older for the years 1995 through 2030 can be modeled by

$$N(x) = 0.03x^2 + 0.32x + 34.23 \qquad \text{million people,}$$

where x is the number of years after 2000.

(a) Write out the rate-of-change formula for the number of Americans age 65 or older.(With **unit**.)

$N'(x) = 0.06x + 0.32$ million people/year

(b) Fill in the following table. Round numerical approximations to two decimal places.

	1995	2011	2030	Units
x	-5	11	30	year
Number of Americans age 65 or older	33.38	41.38	70.83	million people
Rate of change of number of Americans age 65 or older	0.02	0.98	2.12	million people per year