Solution (A)

MATH 1231 Quiz 3 (50pts)
Fall 2014

Name: _____

No calculators are permitted on this test.

1. (10 points) Let $f(x) = 3x^2 - 2x + 5$.

(a) Find the average rate of change of $f(x)$ between the points $(x, f(x))$ and $(x+h, f(x+h))$. Show **all your algebra** and simplify the answer.

$$ARC = \frac{f(x+h)-f(x)}{x+h-x} = \frac{3(x+h)^2-2(x+h)+5-(3x^2-2x+5)}{h}$$

$$= \frac{3x^2+6xh+3h^2-2x-2h+5-3x^2+2x-5}{h} = \frac{6xh+3h^2-2h}{h} = 6x+3h-2$$

(b) Use your answer to part (a) and the **limit definition** to find $f'(x)$. Do **not** use derivative rules.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}(6x+3h-2) = 6x-2$$

2. Use **derivative rules** to find the derivatives: (6 points each)

(a) $h(x) = 5^6 - 5x^6 + 3x^7$

$$h'(x) = -30x^5 + 21x^6$$

(b) $p(t) = 3\sqrt[4]{t^3} - 2e^t + 2\sqrt{t}$ $= 3t^{\frac{3}{4}} - 2e^t + 2t^{\frac{1}{2}}$

$$p'(t) = \frac{9}{4}t^{-\frac{1}{4}} - 2e^t + t^{-\frac{1}{2}}$$

(c) $g(s) = \frac{2}{5s^3} - 2s^{-1.6} + e^2$ $= \frac{2}{5}s^{-3} - 2s^{-1.6} + e^2$

$$g'(s) = \frac{-6}{5}s^{-4} + 3.2s^{-2.6}$$

(A)

(d) $q(x) = 3(2.3)^x - 1.5\ln(x) + 2x^3$

$q'(x) = 3(\ln 2.3)\,2.3^x - \frac{1.5}{x} + 6x^2$

3. Let $C(x)$ be the cost in hundreds of dollars (to a manufacturer) of making x thousand pencils.

(a) (2 points) What are the units of $C'(x)$?

hundreds of dollars per thousand pencils

(b) (3 points) Write a complete sentence with units that gives the practical meaning of the following statement.

$$C(50) = 12$$

When the manufacturer make 50 thousand pencils, the cost will be 12 hundreds of dollars

(c) (5 points) Write a complete sentence with units that gives the practical meaning of the following statement. Do not use words such as per, rate, slope, derivative or any term relating to calculus.

$$C'(50) = 0.23.$$

If the production of the manufacturer increase from 50 to 51 thousand, the cost will increase 0.23 (approx) hundreds dollars

4. Find the slope of the tangent line at $x = 1$ of the function $f(x) = 4^x + x^3$. (6 points)

$f'(x) = (\ln 4)\,4^x + 3x^2$

$f'(1) = (\ln 4)\,4 + 3$

Find a formula for the above tangent line. (2 extra points.)

$f(1) = 5$

$y - 5 = f'(1)\cdot(x - 1)$