

Solutions

A

MATH 1231 Midterm (100 pts) Fall 2014

Instructor: He Wang Name: _____

For full credit, show your work

1. Total federal tax receipts from individuals, in hundreds of billions of dollars, x years after 1989 are approximated in the following table:

x , Years since 1989	1	3	5	7	9	11
Tax Receipts, in hundreds of billions of \$	4.7	4.8	5.4	6.6	8.3	10

- (a) (4 points) Let $T(x)$ be the (total) federal tax receipts from individuals x years after 1989. Use the table above to fit the best model for $T(x)$ from among the choices: EXPONENTIAL or QUADRATIC. Give your model with three decimal places and with units.

$$T(x) = ax^2 + bx + c$$

hundreds of billions of \$

$$a = 0.055$$

$$b = -0.119$$

$$c = 4.706$$

- (b) (4 points) From the model in part (a), derive a function giving the rate of change of federal tax receipts from individuals x years after 1989.

$$T'(x) = 2ax + b = 0.11x - 0.119$$

hundreds of billions of \$ / year

rate of change

- (c) (2 points) According to the model in part (a), what were federal tax receipts from individuals in 1997? Show work and give your answer (to two decimal places) with units.

$$1997 - 1989 = 8$$

$$T'(8) = 0.77 \text{ or } 0.76$$

hundreds of billions of \$ / year

2. Find the derivative of each function. Show as much work as possible. In each case give the exact answer. Do not round any numbers. (6 points each)

(a) $g(x) = \frac{2}{x^6} - 10 \ln(5) + 25e^{2x} - 15\sqrt[5]{x^3} = 2x^{-6} - 10 \ln(5) + 25e^{2x} - 15x^{\frac{3}{5}}$

$$g'(x) = -12x^{-7} + 50e^{2x} - 9x^{-\frac{2}{5}}$$

(b) $j(x) = 14\sqrt{2x^3 - 9 + 8^x} = 14(2x^3 - 9 + 8^x)^{\frac{1}{2}}$

$$j'(x) = 7(2x^3 - 9 + 8^x)^{-\frac{1}{2}} (6x^2 + (\ln 8)8^x)$$

(c) $k(x) = (8x^{-1} - 7 \ln(x))(1.5^x + 3x)$

$$f(x) = 8x^{-1} - 7 \ln x$$

$$f'(x) = -8x^{-2} - \frac{7}{x}$$

$$g(x) = 1.5^x + 3x$$

$$g'(x) = (\ln 1.5)1.5^x + 3$$

$$k'(x) = (8x^{-1} - 7 \ln x) (\ln 1.5)1.5^x + 3 + (1.5^x + 3x) \left(-8x^{-2} - \frac{7}{x}\right)$$

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(d) $m(x) = -9 \ln(7x^8 - 2x^{-2} - e^2)$ $u(x) = 56x^7 + 4x^{-3}$

$$m'(x) = \frac{-9(56x^7 + 4x^{-3})}{7x^8 - 2x^{-2} - e^2}$$

(e) $h(x) = \frac{3x - 2^x}{2\sqrt[3]{x^5}} = (3x - 2^x) \left(\frac{1}{2} \cdot x^{-\frac{5}{3}}\right)$

~~$f(x) = 3x - 2^x$
 $f'(x) = 3 - (\ln 2)2^x$
 $g(x) = \frac{1}{2}x^{-\frac{5}{3}}$
 $g'(x) = -\frac{5}{6}x^{-\frac{8}{3}}$~~

$$h'(x) = (3x - 2^x) \left(-\frac{5}{6}x^{-\frac{8}{3}}\right) + \left(\frac{1}{2}x^{-\frac{5}{3}}\right) \left(3 - (\ln 2)2^x\right)$$

3. From 1975 to 2000, the number of subscribers to basic cable TV (in the U.S.), in millions of subscribers, can be modelled by the function

$$C(x) = \frac{57.6}{1 + 18.4e^{-0.3x}} + 9.7, = 57.6(1 + 18.4e^{-0.3x})^{-1} + 9.7$$

where x is the number of years after 1975.

(a) (2 points) According to the model, what was the number of subscribers to basic cable TV in 1985. Show work. Give your answer (rounded to two decimal places) with units.

$1985 - 1975 = 10$ $C(10) = 39.76$ millions of subscribers

(b) (6 points) Write down a function which gives the rate of change of the number of subscribers to basic cable TV, x years after 1975.

$$C'(x) = -57.6(1 + 18.4e^{-0.3x})^{-2} (18.4)(-0.3)e^{-0.3x}$$

millions of subscribers / year

(c) (2 points) How rapidly was the number of subscribers to basic cable TV changing in 1985? Show work. Give your answer (to two decimal places) with units.

$C'(10) = 4.31$ millions of subscribers / year

$Y_1 = C(x)$
 $C'(10) = nDeriv(Y_1, X, 10)$

(d) (2 points) Explain the meaning of your answer to part (c) in a complete sentence with units. Do not use words like "rate", "per", "derivative" or "slope" or any term relating to calculus.

$= \frac{d}{dx}(Y_1) \Big|_{x=10}$

From 1985 to 1986, the subscribers to basic cable TV increase by approx 4.31 million.

4. (4 points) Ed places \$7,500 into an account at 3.6% interest compounded monthly. Give the formula for the amount $E(t)$ (dollars) in the account after t years.

$$E(t) = 7500 \left(1 + \frac{0.036}{12}\right)^{12t}$$

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5. The balance (in dollars) in Hannah's investment account after t years is given by the formula:

$$A(t) = 4000(1.035)^t$$

Please answer the following questions. Round off all numerical answers in (b) and (c) to two decimal places.

(a) (4 points) Write down the function giving the rate at which the balance is changing after t years.

$$A'(t) = 4000 \ln(1.035) (1.035)^t \quad \$/\text{year}$$

(b) (2 points) How much is the balance after 10 years? Show work. Give your answer with units.

$$A(10) = 5642.40 \quad \$$$

(c) (2 points) At what rate is the balance changing after 10 years? Show work. Give your answer with units.

$$A'(10) = \text{Deriv}(A(t), t, 10) = 194.11 \quad \$/\text{year}$$

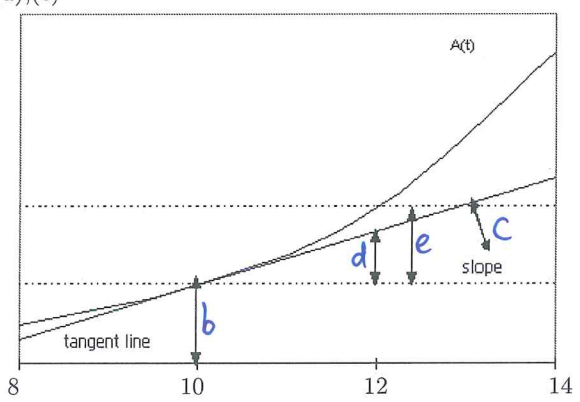
(d) (2 points) Using the result in (c), **estimate** the amount of interest that the account earns from the 10th to the 12th year. Give your answer with units.

~~$$A(12) - A(10) \approx A'(10) (12 - 10) = 194.11 \times 2 = 388.22 \quad \$$$~~

(e) (2 points) Compute the **actual** amount of interest that the account earns from the 10th to the 12th year. Show work. Give your answer with units.

$$A(12) - A(10) = 401.88 \quad \$$$

(f) (2 points) Part of the graph of $A(t)$ and the tangent line at $t = 10$ are shown below. Label the corresponding answer to parts in (b),(c),(d),(e)



6. (6 points) Let $f(x) = 4x^2 - 6x - 3$. Find the **average rate of change** of $f(x)$ between the points $(x, f(x))$ and $(x+h, f(x+h))$. Show all your algebra and simplify your answer.

$$\begin{aligned} \text{ARC} &= \frac{f(x+h) - f(x)}{x+h - x} \\ &= \frac{4(x+h)^2 - 6(x+h) - 3 - (4x^2 - 6x - 3)}{h} \\ &= \frac{4x^2 + 8xh + 4h^2 - 6x - 6h - 3 - 4x^2 + 6x + 3}{h} \\ &= \frac{8xh + 4h^2 - 6h}{h} = 8x + 4h - 6 \end{aligned}$$

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7. The owner of a consumer electronics store has found that the number of HDTVs she sells is modeled by the function:

$$D(x) = 950e^{-0.002x} - 10,$$

where x is the selling price of an HDTV in dollars. Please answer the following questions. Round off numerical answers in (c) and (d) to two decimal places.

- (a) (2 points) Write down a model for $R(x)$, the revenue (in dollars) as a function of price.

$$R(x) = x \cdot D(x) = x(950e^{-0.002x} - 10) \quad \$$$

- (b) (6 points) Write down a formula for the rate of change of revenue in terms of price.

$$R'(x) = (-1.9e^{-0.002x} \cdot x) + (950e^{-0.002x} - 10)$$

$f(x) = x$
 $g(x) = 950e^{-0.002x} - 10$
 $f'(x) = 1$
 $g'(x) = \frac{d}{dx}(950e^{-0.002x} - 10) = -1.9e^{-0.002x}$

- (c) (2 points) When the selling price of an HDTV is \$1500, what is the owner's revenue? Show work. Give your answer with units.

$$R(1500) = 55946.57 \quad \$$$

- (d) (2 points) When the selling price of an HDTV is \$1500, what is the rate of change of revenue? Show work. Give your answer with units.

$$R'(1500) = -104.60 \quad \$/\$$$

8. A company makes staples at a daily cost of $C(x) = 21x(1.126^x) + 120.5$ dollars where x is the number of hundreds of staples produced.

- (a) (4 points) Find the marginal cost function.

$$C'(x) = 21x \ln(1.126)(1.126)^x + (21)(1.126^x)$$

$21x \cdot 1.126^x \cdot \ln(1.126) + 21 \cdot 1.126^x$
 $\$/\text{hundred staples}$

- (b) (2 points) Find the marginal cost of producing 300 staples. Show work. Give your answer with units.

$$C'(3) = 40.65 \quad \$/\text{hundred staples}$$

9. (6 points) Find all the critical points for function $f(x) = 2x^3 - 3.5x^2 + 2x - 3$. Show all steps especially factoring. write down both x and y coordinates for the critical points.

$$f'(x) = 6x^2 - 7x + 2 = 0$$

$$\begin{matrix} 2 & -1 \\ 3 & -2 \end{matrix} x$$

$$(2x-1)(3x-2) = 0$$

$$2x-1=0$$

$$x_1 = \frac{1}{2}$$

$$3x-2=0$$

$$x_2 = \frac{2}{3}$$

$$f(x_1) = -2.625$$

$$f(x_2) = -2.630$$

Solutions B

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- (a) (4 points) Let $T(x)$ be the (total) federal tax receipts from individuals x years after 1989. Use the table above to fit the best model for $T(x)$ from among the choices: EXPONENTIAL or QUADRATIC. Give your model with three decimal places and with units.

$$T(x) = ax^2 + bx + c$$

hundreds of billions of \$

$$a = 0.055$$

$$b = -0.119$$

$$c = 4.706$$

- (b) (4 points) From the model in part (a), derive a function giving the rate of change of federal tax receipts from individuals x years after 1989.

$$T'(x) = 0.11x - 0.119 \text{ hundreds of billions of \$ / year}$$

- (c) (2 points) According to the model in part (a), what was the rate of change of federal tax receipts from individuals in 1998? Show work and give your answer (to two decimal places) with units.

$$1998 - 1989 = 9 \quad T'(9) = 0.88 \text{ or } 0.87 \text{ hundreds of billions of \$ / year}$$

2. Find the derivative of each function. Show as much work as possible. In each case give the exact answer. Do not round any numbers. (6 points each)

(a) $g(x) = \frac{3}{x^6} - 10 \ln(6) + 21e^{2x} - 20\sqrt[5]{x^3} = 3x^{-6} - 10 \ln 6 + 21e^{2x} - 20x^{\frac{3}{5}}$

$$g'(x) = -18x^{-7} + 42e^{2x} - 12x^{-\frac{2}{5}}$$

(b) $j(x) = 12\sqrt{2x^3 - 9 + 8^x} = 12(2x^3 - 9 + 8^x)^{\frac{1}{2}}$

$$j'(x) = 6(2x^3 - 9 + 8^x)^{-\frac{1}{2}} (6x^2 + (\ln 8)8^x)$$

(c) $k(x) = (6x^{-1} - 5 \ln(x))(1.2^x + 4x)$

$$f(x) = 6x^{-1} - 5 \ln(x) \quad f'(x) = -6x^{-2} - \frac{5}{x}$$

$$g(x) = 1.2^x + 4x \quad g'(x) = (\ln 1.2)1.2^x + 4$$

$$k'(x) = (6x^{-1} - 5 \ln(x)) ((\ln 1.2)1.2^x + 4) + (1.2^x + 4x) (-6x^{-2} - \frac{5}{x})$$

$$(d) m(x) = -9 \ln(6x^8 - 2x^{-2} - e^3)$$

$$m'(x) = \frac{-9(48x^7 + 4x^{-3})}{6x^8 - 2x^{-2} - e^3}$$

$$(e) h(x) = \frac{5x - 2^x}{2\sqrt[3]{x^5}} = (5x - 2^x) \left(\frac{1}{2} x^{-\frac{5}{3}}\right)$$

$$f(x) = 5x - 2^x \quad f'(x) = 5 - (\ln 2) 2^x$$

$$g(x) = \frac{1}{2} x^{-\frac{5}{3}} \quad g'(x) = -\frac{5}{6} x^{-\frac{8}{3}}$$

$$h'(x) = (5x - 2^x) \left(-\frac{5}{6} x^{-\frac{8}{3}}\right) + \left(\frac{1}{2} x^{-\frac{5}{3}}\right) (5 - (\ln 2) 2^x)$$

3. From 1975 to 2000, the number of subscribers to basic cable TV (in the U.S.), in millions of subscribers, can be modelled by the function

$$C(x) = \frac{57.6}{1 + 18.4e^{-0.3x}} + 9.7,$$

where x is the number of years after 1975.

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$$1985 - 1975 = 10 \quad C(10) = 39.76 \text{ millions of subscribers}$$

- (b) (6 points) Write down a function which gives the rate of change of the number of subscribers to basic cable TV, x years after 1975.

$$C'(x) = -57.6 (1 + 18.4e^{-0.3x})^{-2} (18.4(-0.3)) e^{-0.3x}$$

millions of subscribers/year

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From 1985 to 1986, the subscribers to basic cable TV increase by approx 4.31 million

4. (4 points) Ed places \$6,500 into an account at 3.5% interest compounded monthly. Give the formula for the amount $E(t)$ (dollars) in the account after t years.

$$E(t) = 6500 \left(1 + \frac{0.035}{12}\right)^{12t}$$

5. The balance (in dollars) in Hannah's investment account after t years is given by the formula:

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- (b) (2 points) How much is the balance after 10 years? Show work. Give your answer with units.

$$A(10) = 5642.40 \text{ \$}$$

- (c) (2 points) At what rate is the balance changing after 10 years? Show work. Give your answer with units.

$$A'(10) = 194.11 \text{ \$/year}$$

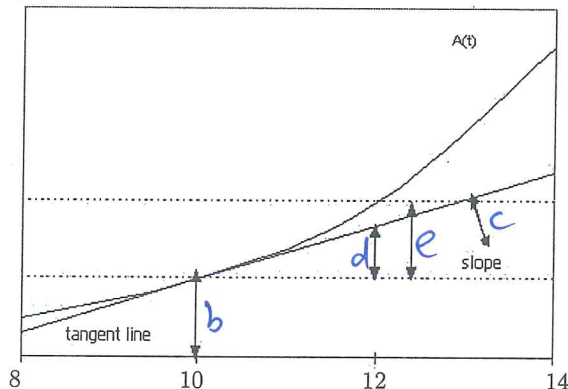
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$$A(12) - A(10) \approx A'(10)(12-10) = 194.11 \times 2 = 388.22 \text{ \$}$$

- (e) (2 points) Compute the actual amount of interest that the account earns from the 10th to the 12th year. Show work. Give your answer with units.

$$A(12) - A(10) = 401.88 \text{ \$}$$

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$$\text{ARC} = \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{4(x+h)^2 - 6(x+h) - 3 - (4x^2 - 6x - 3)}{h}$$

$$= \frac{4x^2 + 8xh + 4h^2 - 6x - 6h - 3 - 4x^2 + 6x + 3}{h}$$

$$= \frac{8xh + 4h^2 - 6h}{h} = 8x + 4h - 6$$

B

7. The owner of a consumer electronics store has found that the number of HDTVs she sells is modeled by the function:

$$D(x) = 950e^{-0.002x} - 10,$$

where x is the selling price of an HDTV in dollars. Please answer the following questions. Round off numerical answers in (c) and (d) to two decimal places.

- (a) (2 points) Write down a model for $R(x)$, the revenue (in dollars) as a function of price.

$$R(x) = x \cdot D(x) = x(950e^{-0.002x} - 10) \quad \$$$

- (b) (6 points) Write down a formula for the rate of change of revenue in terms of price.

$$R'(x) = (-1.9e^{-0.002x})x + (950e^{-0.002x} - 10)$$

$f(x) = x$
 $g(x) = 950e^{-0.002x} - 10$
 $f'(x) = 1$
 $g'(x) = -1.9e^{-0.002x}$

- (c) (2 points) When the selling price of an HDTV is \$1400, what is the owner's revenue? Show work. Give your answer with units.

$$R(1400) = 66877.38 \quad \$$$

- (d) (2 points) When the selling price of an HDTV is \$1400, what is the rate of change of revenue? Show work. Give your answer with units.

$$R'(1400) = -113.99 \quad \$/\$$$

8. A company makes staples at a daily cost of $C(x) = 21x(1.126^x) + 120.5$ dollars where x is the number of hundreds of staples produced.

- (a) (4 points) Find the marginal cost function.

$$C'(x) = 21x(\ln 1.126) 1.126^x + (21)(1.126)^x$$

$\$/\text{hundred staples}$

$21x$
 1.126^x
 21
 $\ln(1.126) 1.126^x$

- (b) (2 points) Find the marginal cost of producing 300 staples. Show work. Give your answer with units.

$$C'(3) = 40.65 \quad \$/\text{hundred staples}$$

9. (6 points) Find all the critical points for function $f(x) = 2x^3 + 3.5x^2 + 2x - 3$. Show all steps especially factoring. write down both x and y coordinates for the critical points.

$$f'(x) = 6x^2 + 7x + 2$$

$$x_1 = -\frac{1}{2} \quad x_2 = -\frac{2}{3}$$

$$\begin{matrix} 2 & 1 \\ 3 & 2 \end{matrix} \times$$

$$f(x_1) = -3.375$$

$$f(x_2) = -3.370$$

$$(2x+1)(3x-2) = 0$$

$$2x+1=0 \quad 3x-2=0$$