Solution

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Q: How to prepare Midterm?
The **main sources** for preparing the Midterm are the **Midterm Review, Quizzes 1-4, QuizReviews 1-4, Class notes (including Handouts), and the following questions**. You should also finish the **WebAssign** in Chapter 2 and Chapter 3, Word problems(I)(II), Factoring, Exponents.

1. The value of (manufacturers') shipments of CD albums, in millions of dollars, x years after 1988 is given by the following table:

x, Years since 1988	2	4	6	8	10	12
Value, in millions of \$.	3.4	6.9	9.1	11	11.9	12.6

a) Let $V(x)$ be the value of shipments of CD albums x years after 1988. Use the table above to fit the best model for $V(x)$ from among the choices - EXPONENTIAL or QUADRATIC. Give your model with three decimal places and with **units**.

$V(x) = y = ax^2 + bx + c$ millions of \$.

$a = -0.086$
$b = 2.099$
$c = -0.340$

b) From the model in part (a), derive a function giving the rate of change of the value of shipments of CD albums x years after 1988.

$V'(x) = -0.172x + 2.099$ millions of \$ per year.

c) According to the model, what was the rate of change of the value of shipments of CD albums in 1995? Show work and give your answer (rounded to 3 decimal places) with units.

$V'(7) = 0.895$ millions \$/year. or $V'(7) = 0.899$ if using $\cdot Y1 = V(x)$ $\cdot$ nDeriv(Y1,X,7)

c) According to the model, what was the the value of shipments of CD albums in 1999? Show work and give your answer (rounded to 3 decimal places) with units.

$V(11) = 12.373$ millions \$ or $V(11) = 12.343$ if you use function in (a)

2. From 1985 to 2010, the number of Mac user (in the U.S.), in millions, can be modeled by the function

$$N(x) = \frac{45.7}{1 + 16.5e^{-0.4x}} + 10.6 = 45.7(1 + 16.5e^{-0.4x})^{-1} + 10.6$$

where x is the number of years after 1985.

a) According to the model, what was the number of Mac user in 1995. Show work. Give your answer (rounded to two decimal places) with units.

$N(10) = 45.69$ millions

b) Write down a function which gives the rate of change of the number of Mac user x years after 1985.

$N'(x) = -45.7(1 + 16.5e^{-0.4x})^{-2}(16.5)(-0.4)e^{-0.4x}$ millions/year

c) How rapidly was the number of Mac user changing in 1995? Show work. Give your answer (to two decimal places) with units.

put $Y1 = N(x)$, $N'(10) = \frac{d}{dx}(Y1)\Big|_{x=10} = 3.26$ millions/year

d) Explain the meaning of your answer in (c) in a complete sentence with units. Do not use words like "rate", "per", "derivative" or "slope".

From 1995 to 1996, the number of Mac user (in the U.S.) increases approx 3.26 millions.

$u(x)$

3. Find the derivative of each function. **(Practice more similar problems in handouts.)**

(a) $f(x) = \dfrac{15}{1 + 21.1e^{-2.3x}} - 2x^5$ $= 15\left(1+21.1e^{-2.3x}\right)^{-1} - 2x^5$

$$f'(x) = -15\left(1+21.1e^{-2.3x}\right)^{-2} \cdot (21.1)(-2.3)e^{-2.3x} - 10x^4$$

(b) $g(x) = \dfrac{6}{x^5} + 15e^x - 25\sqrt[5]{x^4}$ $= 6x^{-5} + 15e^x - 25x^{\frac{4}{5}}$

$$g'(x) = -30x^{-6} + 15e^x - 20x^{-\frac{1}{5}}$$

$u(x)$

(c) $j(x) = 12\sqrt{x^7 - e^2 + 7^x}$ $= 12\left(x^7 - e^2 + 7^x\right)^{\frac{1}{2}}$

$$j'(x) = 12 \cdot \left(\tfrac{1}{2}\right)\left(x^7 - e^2 + 7^x\right)^{-\frac{1}{2}}\left(7x^6 + (\ln 7)7^x\right)$$

(d) $k(x) = (8x^2 - 3\ln(x))(0.5^x)$

$f(x) = 8x^2 - 3\ln x$ $\quad f'(x) = 16x - \frac{3}{x}$

$g(x) = 0.5^x$ $\quad g'(x) = \ln(0.5) \cdot 0.5^x$

$$k'(x) = \left(8x^2 - 3\ln x\right)\left(\ln(0.5) \cdot 0.5^x\right) + 0.5^x\left(16x - \frac{3}{x}\right)$$

(e) $m(x) = -9\ln(11x^3 - 2x^{-1} - 10)$

$u(x)$

$$m'(x) = \frac{-9}{11x^3 - 2x^{-1} - 10} \cdot \left(33x^2 + 2x^{-2}\right)$$

4. (a)If Jimmy places \$5000 into an account at 2.05% interest compounded quarterly. Give the formula for the amount $A(t)$ (dollars) in the account after t years.
(b)If Jimmy places \$9000 into an account at 3.05% interest compounded continuously. Give the formula for the amount $B(t)$ (dollars) in the account after t years. **(Practice more similar questions in ClassPacket page 43.)**

(a) $A(t) = 5000 \cdot \left(1 + \dfrac{0.0205}{4}\right)^{4t}$ \$ $\qquad B(t) = 9000 \cdot e^{0.0305t}$ \$

5. Let $f(x) = -3x^2 - 7x + 10$. Find the average rate of change of $f(x)$ between the points $(x,\ f(x))$ and $(x+h,\ f(x+h))$. Show **all your algebra** and **simplify your answer.** NO Derivative!

$$ARC = \frac{f(x+h) - f(x)}{x+h-x} = \frac{-3(x+h)^2 - 7(x+h) + 10 - (-3x^2 - 7x + 10)}{h}$$

$$= \frac{-3x^2 - 6xh - 3h^2 - 7x - 7h + 10 + 3x^2 + 7x - 10}{h} = \frac{-6xh - 3h^2 - 7h}{h} = -6x - 3h - 7$$

6. The balance (in dollars) in Mike's bank account after t years is given by the formula: $A(t) = 2000(1.065)^t$. Please answer the following questions. Round off all numerical answers in (b), (c), and (d) to two decimal places.

a) Write down the function giving the rate at which the balance is changing after t years.

$A'(t) = 2000 \ln(1.065)(1.065^t)$ $/year

b) How much is the balance after 12 years? Show work. Give your answer with units.

$A(12) = 4258.19$ $

c) At what rate is the balance changing after 12 years? Show work. Give your answer with units.

$A'(12) = 268.16$ $/year

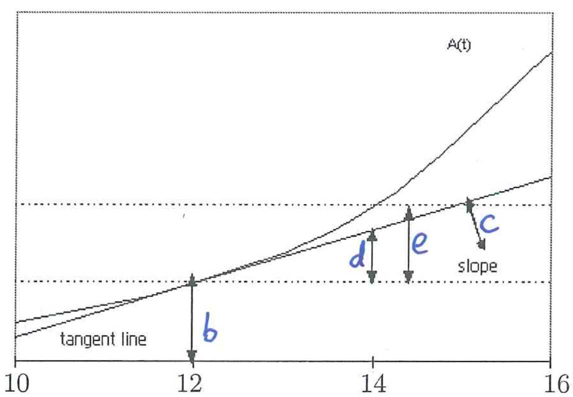
d) Using the result in (c), **estimate** the amount of interest that the account earns from the 12th to the 14th year.

$h = 14 - 12 = 2$ $\quad A(14) - A(12) \approx 2f'(12) = 2 \times 268.16 = 536.32$ $

e) Compute the **actual** amount of interest that the account earns from the 12th to the 14th year. Show work. Give your answer with units.

$A(14) - A(12) = 571.56$ $

(f) Part of the graph of $A(t)$ and the tangent line at $t = 12$ are shown below. Label teh corresponding answer to parts in (b),(c),(d),(e)



7. Find all the critical points for function $f(x) = 4x^3 - 5.5x^2 + 2x - 5$. Show all steps especially factoring. write down both x and y coordinates for the critical points. **(Practice factoring problems in page 42 in ClassPackage.)**

Solve $f'(x) = 0$

$12x^2 - 11x + 2 = 0$

Factor: $\begin{matrix} 3 & & -2 \\ 4 & X & -1 \end{matrix}$

$(3x - 2)(4x - 1) = 0$

$3x - 2 = 0 \quad 4x - 1 = 0$

$x_1 = \frac{2}{3} \quad x_2 = \frac{1}{4}$ ---- x coordinates

$f(x_1) = f(\frac{2}{3}) = -4.926$ --- y coordinates

$f(x_2) = f(\frac{1}{4}) = -4.781$

8. The owner of electronics store has found that the number of computers he sells is modelled by the function:

$$D(x) = 950e^{-0.002x} - 10,$$

where x is the selling price of a ~~ipad~~ computer in dollars.

Please answer the following questions. Round off numerical answers in (c), and (d) to two decimal places.

a) Write down a model for $R(x)$, the revenue (in dollars) as a function of price.

$$R(x) = x \cdot D(x) = x(950e^{-0.002x} - 10) \quad \$$$

b) Write down a formula for the rate of change of revenue in terms of price.

product rule $$R'(x) = (950e^{-0.002x} - 10) + x(950(-0.002)e^{-0.002x}) \quad \$/\$$$

c) When the selling price of a computer is \$800, what is the owner's revenue? Show work. Give your answer with units.

$$R(800) = 14541.35 \quad \$$$

d) When the selling price of a computer is \$800, what is the rate of change of revenue? Show work. Give your answer with units.

$$R'(800) = -125.08 \quad \$/\$$$

e) If each computer costs the owner \$320, and the owner has no other costs for selling the computers, write down a model for $P(x)$, the profit (in dollars) as a function of price.

$$P(x) = (x-320)D(x) = (x-320)(950e^{-0.002x} - 10) \quad \$$$

9. A company makes golf ball at a daily cost of $C(x) = 26x(1.125^x) + 150.5$ dollars where x is the number of hundreds of golf ball produced.

(a) Find the marginal cost function.

$$C'(x) = 26(1.125^x) + 26x \cdot \ln(1.125)(1.125^x) \quad \$/\text{hundreds of golf ball}$$

(b) Find the marginal cost of producing ~~1~~ 200 golf ball. Show work. Give your answer with units.

$$C'(2) = 40.66 \quad \$/\text{hundred} \cdot \text{golf balls}$$

(c) Assume that it costs \$216.3 to produce 200 golf balls. Use this information and your answer to part (b) to **estimate** the cost of producing 203 golf balls. Show work, with units.

$$C(2) = 216.3 \ \$$$

$$C(2.03) \approx C(2) + 0.03\,C'(2) = 217.52 \ \$$$

(d) Find the **accurate** cost of produce producing 203 golf balls.

$$C(2.03) = 217.54 \ \$$$