1. Chain Rule for Derivative.

> If $h(x)$ is a composition function $f(u(x))$, then
>
> $$h'(x) = f'(u) \cdot u'(x)$$

Example: Compute the derivative of $\cancel{f(x) = \frac{3}{(4x^5 - \sqrt[3]{x^4})^4} + x^5}$ $u(x) = \ln(3e^x + 2x^3)$

$$\begin{cases} s(x) = 3e^x + 2x^3 \\ g(s) = \ln s \end{cases}$$

$$\begin{cases} s'(x) = 3e^x + 6x^2 \\ g'(s) = \frac{1}{s} \end{cases}$$

$$u'(x) = g'(s) \cdot s'(x)$$
$$= \frac{1}{s} \cdot (3e^x + 6x^2)$$
$$= \frac{3e^x + 6x^2}{3e^x + 2x^3}$$

2. Using the Chain rule twice.

Example: Compute the derivative of $f(x) = \dfrac{3}{(\ln(3e^x + 2x^3))^4}$

$$\begin{cases} u(x) = \ln(3e^x + 2x^3) \\ h(u) = \frac{3}{u^4} = 3 \cdot u^{-4} \end{cases}$$

$$\begin{cases} u'(x) = \frac{3e^x + 6x^2}{3e^x + 2x^3} \\ h'(u) = -12u^{-5} \end{cases}$$

$$f'(x) = h'(u) \cdot u'(x)$$
$$= -12u^{-5} \cdot \left(\frac{3e^x + 6x^2}{3e^x + 2x^3}\right)$$
$$= -12\left(\ln(3e^x + 2x^3)\right)^{-5}\left(\frac{3e^x + 6x^2}{3e^x + 2x^3}\right)$$

Example: Compute the derivative of $f(x) = 20(-2x^{-2} + 3e^{5x^3})^6$

$$\begin{cases} u(x) = -2x^{-2} + 3e^{5x^3} \\ g(u) = 20u^6 \end{cases}$$

$$\begin{cases} u'(x) = 4x^{-3} + 3e^{5x^3} \cdot 15x^2 \\ g'(u) = 120u^5 \end{cases}$$

$$\begin{cases} s(x) = 5x^3 \\ h(s) = 3e^s \end{cases} \quad \begin{cases} s'(x) = 15x^2 \\ h'(s) = 3e^s \end{cases}$$

$$\left(3e^{5x^3}\right)' = 3e^{5x^3} \cdot 15x^2$$

$$f'(x) = g'(u) \cdot u'(x)$$
$$= 120(-2x^{-2} + 3e^{5x^3})^5(4x^{-3} + 3e^{5x^3} \cdot 15x^2)$$

3. Chain rule for logistic model

In quiz2, we have a Logistic model for a cost function $C(x)$ (thousands of dollars) respect to the number of forklifts

$$C(x) = \frac{19.171}{1 + 27.472e^{-0.578x}}$$

Question: What is the rate of change of the cost $C(x)$.

Step1: $u(x) = 1 + 27.472e^{-0.578x}$

$f(u) = \frac{19.171}{u} = 19.171u^{-1}$

Step2: $u'(x) = 27.472(-0.578)e^{-0.578x}$

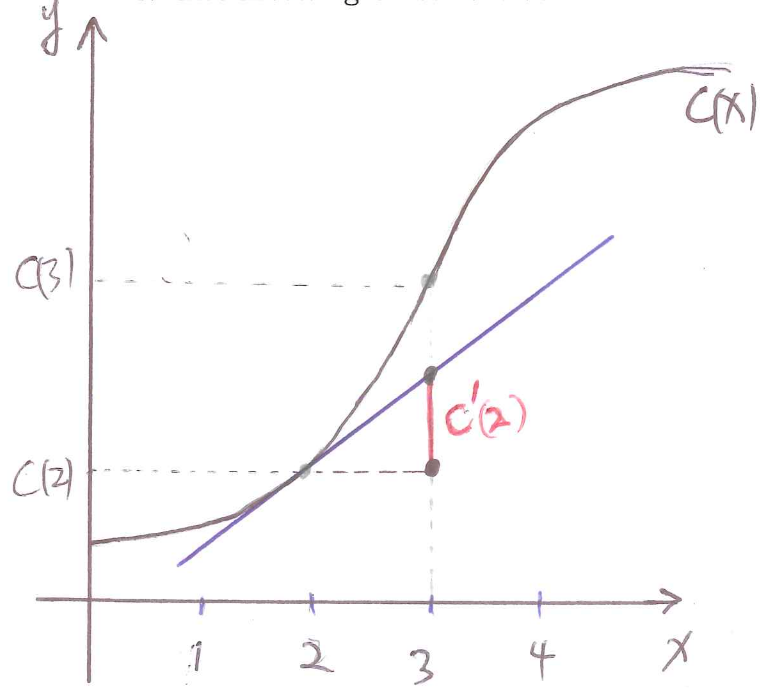
$f'(u) = -19.171u^{-2}$

Step3: $C'(x) = f'(u)\,u'(x)$

$= -19.171u^{-2}(27.472)(-0.578)e^{-0.578x}$

$= -19.171(1 + 27.472e^{-0.578x})^{-2}(27.472)(-0.578)e^{-0.578x}$

4. The meaning of derivative



$C'(2) = 1.03$

$C(3) - C(2) \approx C'(2) = 1.03$

If the production of forklifts increase from 2 to 3,

the cost will increase by approx 1.03 thousands dollars