

• project

• EWA

In the last class

Derivative Operations:

★ If  $f(x) = c \cdot g(x)$  then  $f'(x) = c \cdot g'(x)$  ..... Constant Multiplier rule

★ If  $h(x) = f(x) + g(x)$  then  $h'(x) = f'(x) + g'(x)$  ..... Sum rule

★ If  $h(x) = f(x) - g(x)$  then  $h'(x) = f'(x) - g'(x)$  ..... Difference rule

Derivative of classical functions:

★ If  $f(x) = b$  then  $f'(x) = 0$  ..... Constant rule

★ If  $f(x) = x^n$  then  $f'(x) = nx^{n-1}$  ..... Power rule

We learn some more formulas (rules) today.

★ If  $f(x) = b^x$  then  $f'(x) = (\ln b) \cdot b^x$  ..... Exponential rule

▷▷ If  $f(x) = e^{ax}$  then  $f'(x) = ae^{ax}$       ▷▷ If  $f(x) = e^x$  then  $f'(x) = e^x$

★ If  $f(x) = \ln x$  then  $f'(x) = \frac{1}{x}$  ..... Natural Logarithm rule

Compute the Derivative of the following functions.

Example 1.  $f(x) = 3(2^x)$

$$f'(x) = 3(\ln 2) 2^x$$

Example 2.  $f(x) = 3.01 + 5 \ln x$

$$f'(x) = \frac{5}{x}$$

Example 3.  $f(x) = 2.4 \ln x + e^5$

$$f'(x) = \frac{2.4}{x}$$

Example 4.  $f(x) = 2x + \frac{2}{\sqrt[4]{x^3}} + \pi^2$

$$f'(x) = 2 + 2 \left(-\frac{3}{4}\right) x^{-\frac{7}{4}}$$

Example 5.  $f(x) = \frac{4}{x^\pi} + 5.1 \ln x + 3(5^x)$

$$f'(x) = 4(-\pi) x^{-\pi-1} + \frac{5.1}{x} + 3 \ln(5) \cdot 5^x$$

Example 6.  $f(x) = 5.6(e^x) + 3x^e + 5$

$$f'(x) = 5.6e^x + 3e x^{e-1}$$

**Example:** Let  $P(x)$  be the annual profit in millions of dollars of a store of selling  $x$  hundreds cars.

1. What are the units of  $P(x)$ ?

millions of dollars

2. What are the units of  $P'(x)$ ?

millions of dollars per hundreds of cars

3. Write a complete sentence with units that gives the practical meaning of the following statement.

$$P(30) = 5$$

When the store sell 30 hundreds cars, the annual profit is 5 million <sup>dollars</sup>

4. Write a complete sentence with units that gives the practical meaning of the following statement. Do not use words such as per, rate, slope, derivative or any term relating to calculus.

$$P'(30) = -0.31.$$

If the annual sale increase from 30 hundreds to 31 hundreds, then the annual profit would decrease by approx 0.31 million <sup>dollars</sup>

**Example:** Find the slope of the tangent line at  $x = 2$  of the function  $f(x) = 5^x + 2x^2$ .

$$f'(x) = (\ln 5) \cdot 5^x + 4x$$

$$f'(2) = (\ln 5) 5^2 + 8 \leftarrow \text{the slope of the tangent line at } x=2$$

Find the formula for above the tangent line.

$$f(2) = 5^2 + 2 \cdot 2^2 = 33$$

$$\text{Formula for the tangent line: } y - 33 = f'(2)(x - 2)$$

Ex: Find the slope of the tangent line at  $x=0$  of  $f(x) = 2^x + x^4 + 3$   
 Find a formula for the tangent line

$$f'(x) = (\ln 2) 2^x + 4x^3$$

$$f(0) = 1 + 3 = 4$$

$$y - 4 = (\ln 2)(x)$$

$$f'(0) = \ln 2$$

Exercise1:  $g(x) = 4^3 - 12x^3 + 3x^{40}$

$$g'(x) = -36x^2 + 120x^{39}$$

Exercise2:  $f(x) = 2x^{-1.1} + 25x^4 + e^4$

$$f'(x) = -2.2x^{-2.1} + 100x^3$$

Exercise3.  $f(x) = 8\sqrt[5]{x^3} + 4(\sqrt{x^3}) - \frac{2}{x}$

$$f(x) = 8x^{\frac{3}{5}} + 4x^{\frac{3}{2}} - 2x^{-1}$$

$$f'(x) = \frac{24}{5}x^{-\frac{2}{5}} + 6x^{\frac{1}{2}} + 2x^{-2}$$

Exercise4.  $g(t) = 5\sqrt[3]{t^4} + 2.2(\sqrt{t^5}) - \frac{1}{t^2}$

$$g(t) = 5t^{\frac{4}{3}} + 2.2t^{\frac{5}{2}} - t^{-2}$$

$$g'(t) = \frac{40}{3}t^{\frac{1}{3}} + 5.5t^{\frac{3}{2}} + 2t^{-3}$$

Exercise5.  $h(t) = 5(e^{3t}) - 4 \ln x + \pi$

$$h'(t) = 15e^{3t} - \frac{4}{x}$$

Exercise6.  $f(x) = 5(2^x) - 3 \ln x + e^2$

$$f'(x) = 5(\ln 2)2^x - \frac{3}{x}$$

Exercise7:  $g(x) = \frac{2}{\sqrt[4]{x^3}} - \frac{x}{\sqrt[3]{x^2}}$

$$g(x) = 2x^{-\frac{3}{4}} - x^{\frac{1}{3}}$$

$$g'(x) = -\frac{3}{2}x^{-\frac{7}{4}} - \frac{1}{3}x^{-\frac{2}{3}}$$

Exercise8:  $f(x) = \frac{1}{\sqrt[3]{x}} - \sqrt{x} - 4x$

$$f(x) = x^{-\frac{1}{3}} - x^{\frac{1}{2}} - 4x$$

$$f'(x) = \frac{1}{3}x^{-\frac{4}{3}} - \frac{1}{2}x^{-\frac{1}{2}} - 4$$

Exercise9.  $f(x) = 3x^2(2x - 2.2x^{1.2})$

$$= 6x^3 - 6.6x^{3.2}$$

$$f'(x) = 18x^2 - 6.6 \times 3.2x^{2.2}$$

Exercise10.  $f(x) = 2x^3(5x^2 - 7(\sqrt{x}))$

$$= 10x^5 - 14x^{\frac{7}{2}}$$

$$f'(x) = 50x^4 - \frac{98}{2}x^{\frac{5}{2}}$$

Exercise11.  $f(x) = 4.2x^{\ln 2} + 2x^4 + e^3$

$$f'(x) = 4.2(\ln 2)x^{(\ln 2)-1} + 8x^3$$

Exercise12.  $g(x) = 3x^{\ln 5} + x^e + \pi^3$

$$g'(x) = 3(\ln 5)x^{(\ln 5)-1} + e \cdot x^{e-1}$$

Ex:  $\frac{3x^8 + 10x^7 + 2}{x^3}$