• projects
• WebAssign
• Math 1130

{ practice problems
page 11 in ClassPacket }

• If we have the algebra of $f(x)$, we can compute the **derivative** by the limits definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

In Lecture 4, we computed the example.

► **Example:** Let $f(x) = 1 - 2x - 3x^2$.

(a) Find the average rate of change of $f(x)$ between the points $(x, f(x))$ and $(x+h, f(x+h))$

(b) Find $f'(x)$ using the limit definition and the answer (a)

Now, we do the same computations (a) and (b) for some easier examples.

Example 1: $f(x) = b$.

(a) $ARC = \frac{f(x+h) - f(x)}{h} = \frac{b-b}{h} = 0$

(b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 0$

Example 2: $f(x) = x$.

(a) $ARC = \frac{f(x+h) - f(x)}{h} = \frac{x+h-x}{h} = 1$

(b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 1$

Example 3: $f(x) = x^2$

(a) $ARC = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x+h$

(b) $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} (2x+h) = 2x$

• The formula for the derivative of constant function

★ $\boxed{\text{If } f(x) = b \text{ then } f'(x) = 0}$ .... Constant rule

• The formula for the derivative of power function

★ $\boxed{\text{If } f(x) = x^n \text{ then } f'(x) = nx^{n-1}}$ .... Power rule

Example: $f(x) = x^6$

$f'(x) = 6x^5$

Example: $f(x) = x^{-0.3}$

$f'(x) = -0.3x^{-1.3}$

Example: $f(x) = x^e$

$f'(x) = e \cdot x^{e-1}$

Example: $f(x) = \sqrt{x^3} = X^{\frac{3}{2}}$ **Example**: $f(x) = \dfrac{1}{\sqrt[3]{x^2}} = X^{-\frac{2}{3}}$

$f'(x) = \frac{3}{2}X^{\frac{1}{2}}$ $f'(x) = -\frac{2}{3}X^{-\frac{5}{3}}$

- The formula for the derivative of constant multiply a function.

★ $\boxed{\text{If } f(x) = c \cdot g(x) \text{ then } f'(x) = c \cdot g'(x)}$ ········· Constant Multiplier rule

Example: $f(x) = 2x^9$ **Example**: $f(x) = 3(\sqrt{x}) = 3 \cdot X^{\frac{1}{2}}$

$f'(x) = 2 \times 9 \times X^8 = 18X^8$ $f'(x) = \frac{3}{2}X^{-\frac{1}{2}}$

- The formulas for the derivative of sums and differences of functions.

★ $\boxed{\text{If } h(x) = f(x) + g(x) \text{ then } h'(x) = f'(x) + g'(x)}$ ············ Sum rule

★ $\boxed{\text{If } h(x) = f(x) - g(x) \text{ then } h'(x) = f'(x) - g'(x)}$ ······· Difference rule

Example: $f(x) = 1 - 2x - 3x^2$

$f'(x) = -2 - 6X$

Example: $f(s) = \pi + \dfrac{2}{s^2} - 2.1s^5 = \pi + 2S^{-2} - 2.1S^5$

$f'(s) = -4S^{-3} - 10.5S^4$

Example: $g(x) = \dfrac{3}{\sqrt[4]{x^3}} - 5\sqrt[3]{x^2} = 3 \cdot X^{-\frac{3}{4}} - 5X^{\frac{2}{3}}$

$g'(x) = -\frac{9}{4}X^{-\frac{7}{4}} - \frac{10}{3}X^{-\frac{1}{3}}$

Example: $f(x) = 3\sqrt[3]{x^4} + 4x^\pi + 3x = 3X^{\frac{4}{3}} + 4X^\pi + 3X$

$f'(x) = 4X^{\frac{1}{3}} + 4\pi X^{\pi-1} + 3$

Example : $f(u) = 3u^3 + \dfrac{2}{u} + \dfrac{3}{\sqrt{u^3}}$

$= 3u^3 + 2u^{-1} + 3 \cdot u^{-\frac{3}{2}}$

$f'(u) = 9u^2 - 2u^{-2} - \frac{9}{2}u^{-\frac{5}{2}}$

Example : $h(x) = 3.2x^{\ln 2} + 2x^e + e^2$

$h'(x) = 3.2(\ln 2)X^{(\ln 2)-1} + 2eX^{e-1}$