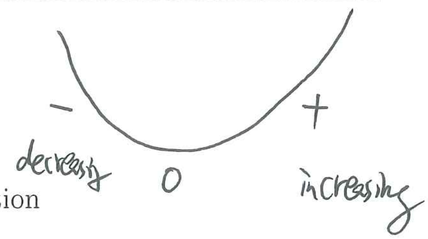
• Percentage rate of change at $x_0 = \frac{f'(x_0)}{f(x_0)} \cdot 100\%$ unit: % per unit of x

1. The Slope Graph of a function



Slope Graph is the graph of the new function $f'(x)$.

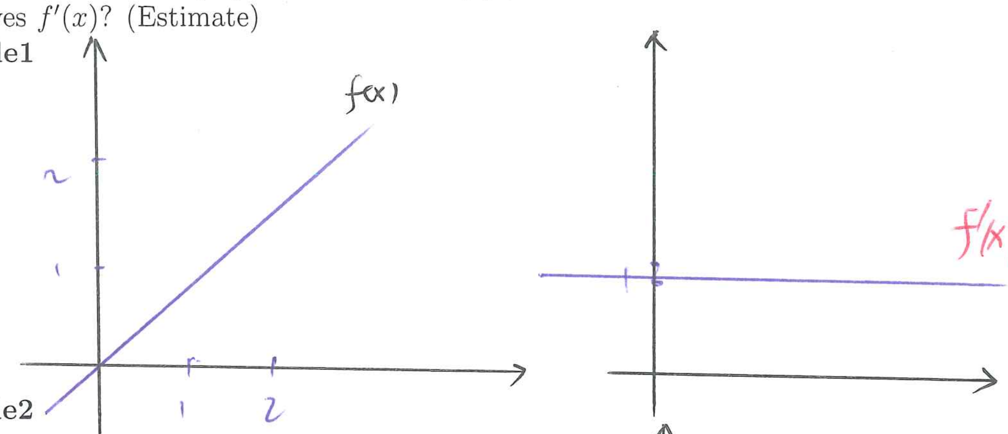
• If we have the algebra of $f(x)$, we can use the limits definition

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

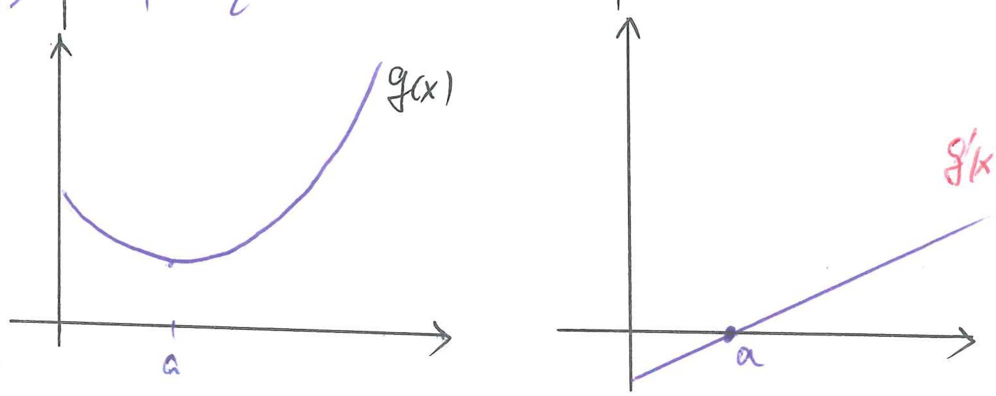
computing $f'(x)$, then find the graph precisely.

• If we only have the graph of a function $f(x)$, how to find the information of all the derivatives $f'(x)$? (Estimate)

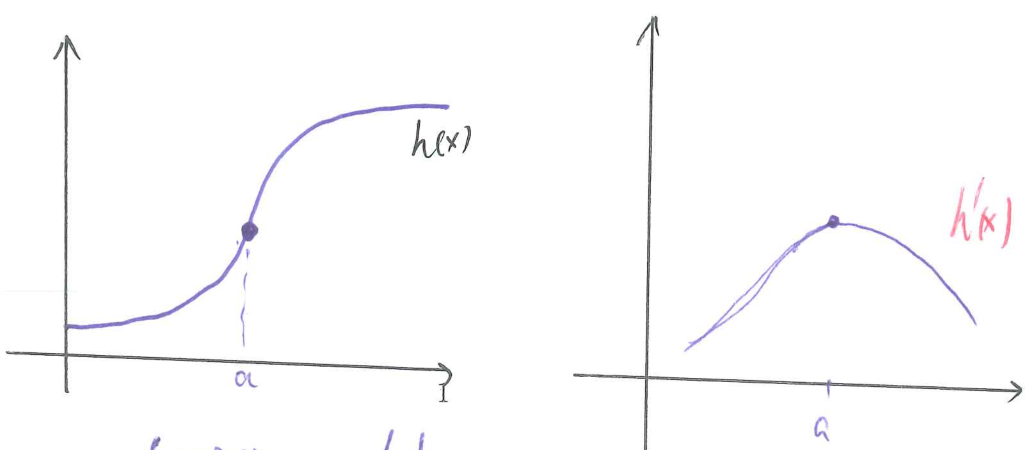
Example1



Example2

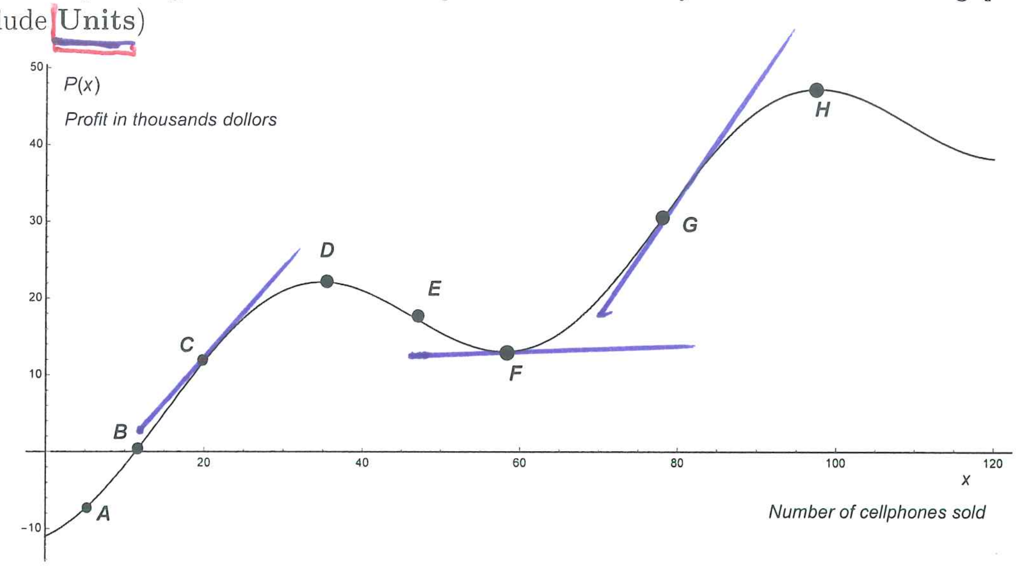


Example3



Logistic model

1. Example The figure gives the average monthly profit for a cellphone store in the last year corresponding to the number of cellphones sold monthly. Answer the following questions (include Units)



Q1: Lable the positive(+), negative (-), or zero(0) for the (instantaneous) rate of change at

A **+** B **+** C **+** D **0** E **−** F **0** G **+** H **0**

Q2: Draw tangent lines at point C, at point F and at point G.

Q3: Lable the positive(+), negative (-), or zero(0) for the average rate of change between two points.

$A \to C$ **+** $D \to F$ **−** $C \to G$ **+**

Q4: When the 20 cellphones sold monthly, what is the profit of the store?

12 thousand dollars

Q5: When the number of cellphones sold from change from 20 to 80, what is the change, percentage change, and average rate of change?

$(20, 12)$ $(80, 32)$
x_0 $f(x_0)$ x_1 $f(x_1)$

Change $= f(x_1) - f(x_0) = 32 - 12 = 20$ thousand \$

percentage change $= \frac{32-12}{12} 100\% = 166.667\%$

$ARC = \frac{32-12}{80-20} = \frac{20}{60} = 0.333$ thousand \$ per cellphones

Q6: Are the change, the percentage change, the average rate of change have the same sign(+, -, or 0)? In Q5

Yes

Q7: Where are the inflection points? Change of concavity concave up down

B, E, G

Q8: Estimate the specific values of number of cellphone sold monthly, for which the derivative of the profit is zero.

35 60 100

Q9: At least how many cellphones the store should sell monthly, if it want to earn money?

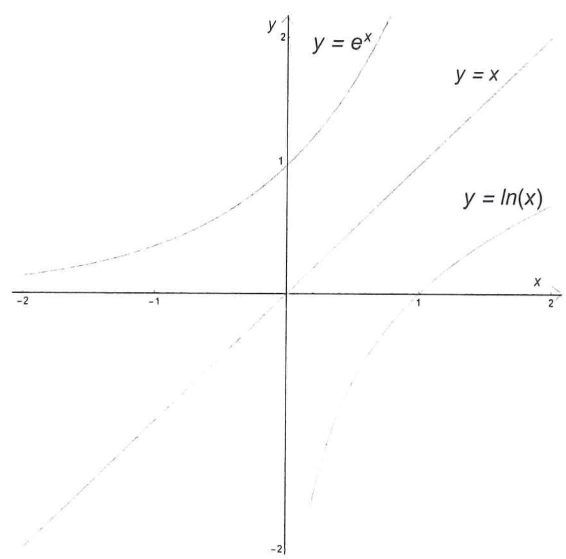
11 cellphones

2. Powers and Logarithms (Page 41 in the ClassPacket.)

$\boxed{x^a \cdot x^b = x^{a+b}}$ $\boxed{\dfrac{x^a}{x^b} = x^{a-b}}$ $\boxed{(x^a)^b = x^{ab}}$ $\boxed{x^{-a} = \dfrac{1}{x^a}}$ $\boxed{\sqrt[n]{x} = x^{\frac{1}{n}}}$ $\boxed{\sqrt[n]{x^a} = x^{\frac{a}{n}}}$

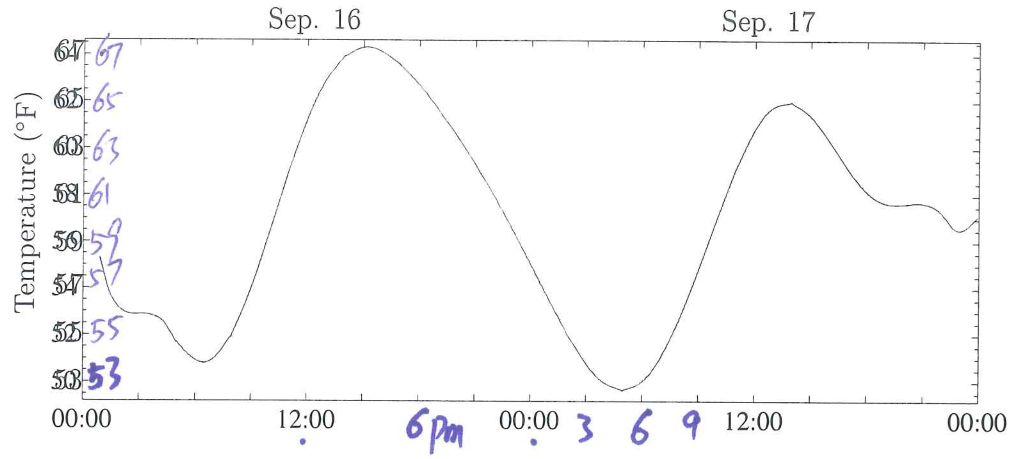
$\boxed{\ln(e^k) = k}$ $\boxed{e^{\ln(k)} = k}$. $\boxed{\ln(ab) = \ln(a) + \ln(b)}$ $\boxed{\ln\left(\dfrac{a}{b}\right) = \ln(a) - \ln(b)}$ $\boxed{\ln(a^k) = k\ln(a)}$

$\boxed{\ln(e) = 1}$ and $\boxed{\ln(1) = 0}$.



The graphs of $y = e^x$ and $y = \ln(x)$ are mirror reflections along the line $y = x$.

The following graph shows the temperature in Boston last Friday and Saturday, in degrees Fahrenheit.



1. Between noon and midnight September 16, the percentage change in temperature was:
positive negative zero (circle one)

2. Around 10 am September 17, the instantaneous rate of change of temperature was:
positive negative zero (circle one)

3. At 6 pm September 16, the temperature was:
increasing decreasing neither (circle one)

4. Estimate and list the specific times for which the derivative of the temperature is zero.

Step 16 : 6am , 2pm

Step 17 : 4 am , 1 pm

5. Estimate the temperature at 6 am September 17. Give units.

53 °F

6. List the approximate location of two inflection points.

10am 9pm · Step 16

9 am step 17

Examples: Simplify the following formulas. No radical , No x in denominator

1. $5(x^{-3})^3 - 8\left(\frac{1}{x^4}\right)^{-2} + \frac{1}{3x^2}$ $= 5x^{-9} - 8x^{8} + \frac{1}{3}x^{-2}$

2. $3\sqrt{x^3} - 4\sqrt[3]{x^2} - \frac{x}{2x^{-4}}$ $= 3x^{\frac{3}{2}} - 4x^{\frac{2}{3}} - \frac{1}{2}x^5$

3. $4(\sqrt[5]{x})^4 + \frac{x}{\sqrt[4]{x}}$ $= 4x^{\frac{4}{5}} + x^{\frac{3}{4}}$

4. $-x^6 - x(3x^{-3} - x^5)$ $= -3x^{-2}$

5. $\frac{x^2 - 34x^5 - 12}{2x^4}$ $= \frac{1}{2}x^{-2} - 17x - 6x^{-4}$

6. $\left(\sqrt{x^2} - 3\left(\frac{2}{x}\right)^{-3} + \frac{x}{3x^{-2}}\right)\ln(1)$ $= 0$

7. $\ln(e^{-x^2}) + e^{\ln(x^2)}$ $= -x^2 + x^2 = 0$

8. $3\left(\sqrt[3]{x^4}\right)^6 + \frac{x}{\sqrt[4]{x}}$ $= 3x^8 + x^{\frac{3}{4}}$

9. $1 - (3x-1)(x+3) - (x+2)^2$ $= -4x^2 - 12x$

10. $\frac{32x^{-3} - 14x^2 - 24x^{-1}}{4x^{-2}}$ $= 8x^{-1} - \frac{7}{2}x^4 - 6x$

Ex: Page 11 in Class packet.