- Instructor: He Wang

Review: Let $f(x)$ be a function. x_0 and x_1 are two inputs.

$$\boxed{\text{change} = f(x_1) - f(x_0)}$$

The unit of change = the unit of $f(x)$.

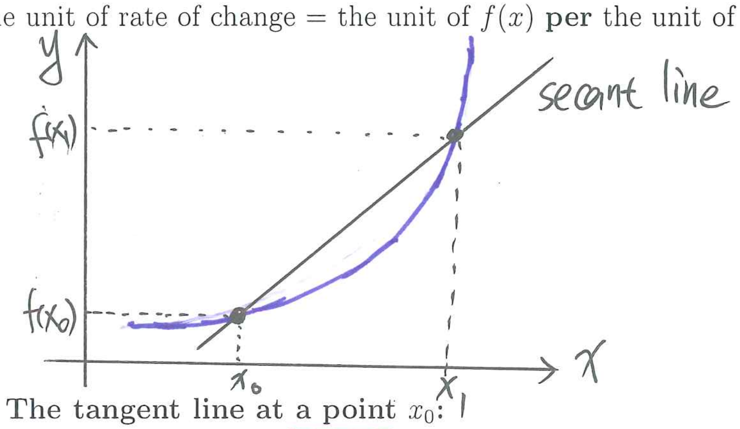
$$\boxed{\text{percentage change} = \frac{f(x_1) - f(x_0)}{f(x_0)} \cdot (100\%)}$$

The unit of percentage change = %

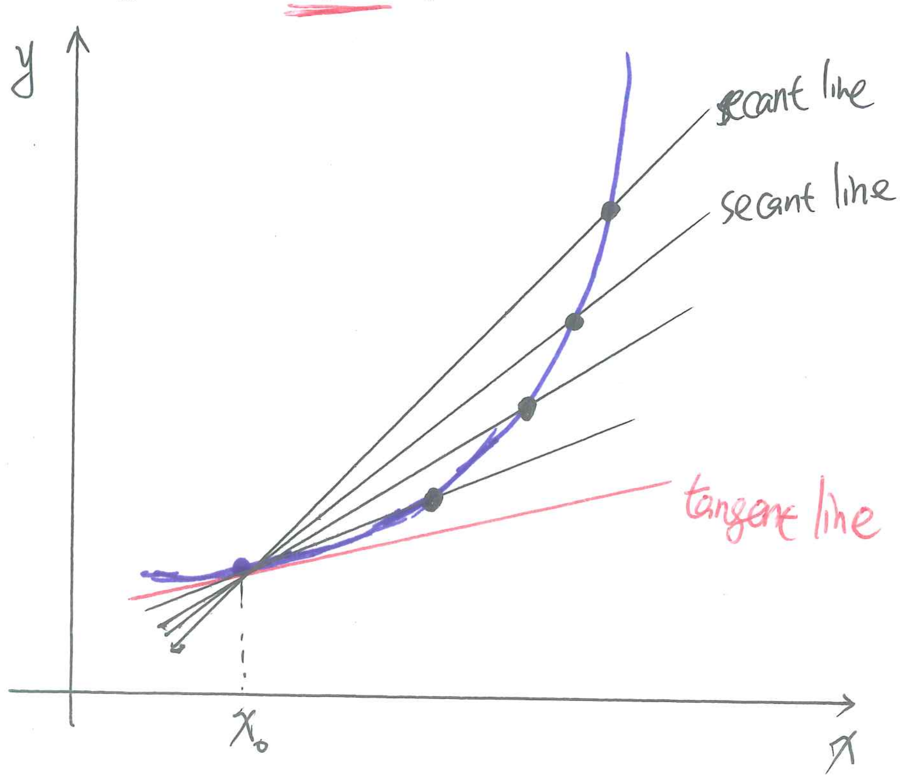
1. The secant line between two points $x_0 < x_1$:

$$\boxed{\text{average rate of change (ARC)} = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \text{slope of the } \textbf{secant line}}$$

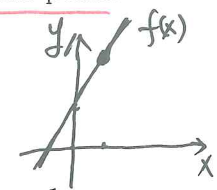
The unit of rate of change = the unit of $f(x)$ **per** the unit of x.



2. The tangent line at a point x_0:



• **Q:** *How to find a tangent line?*
Roughly speaking, the tangent line at x_0 is the limit of the secant line x_0 and x_1, when x_1 is near x_0 very much.
• The ***instantaneous rate of change*** measures the change occurring at a specific point.
• **Q:** Why do we study the tangent line?



$$\boxed{\text{Instantaneous Rate of Change (IRC)} = \text{Slope of the } \textbf{tangent line}}$$

Example 1: $f(x) = 3x + 4$. What is the slope of the tangent line at $x_0 = 1$?

$\frac{f(2)-f(1)}{2-1} = \frac{4-1}{1} = 3$

Example 2: Let $f(x) = x^2$. Estimate the **slope** of the **tangent line** at point $x_0 = 1$.

One

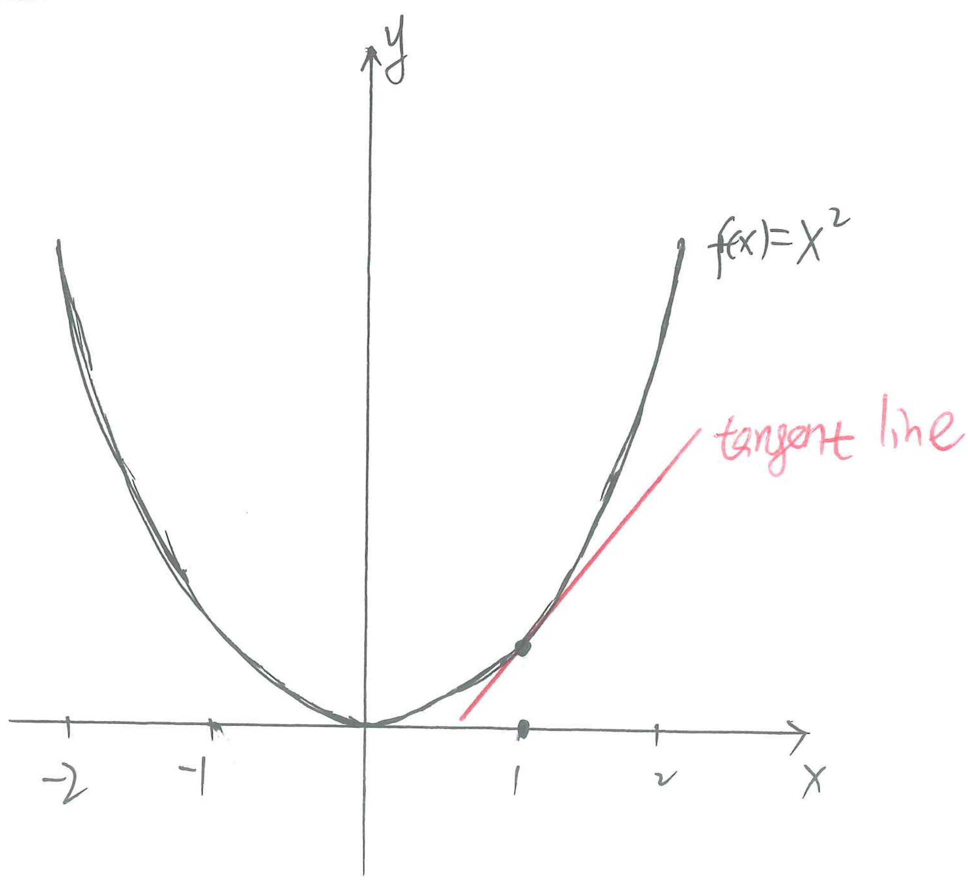
Solution: By the definition, we need to choose a point x_1 near x_0 very much. So, we choose $x_1 = 1.1$. Then the slope of the tangent line at $x_0 = 1$ can be estimated by the slope to the secant line between $x_0 = 1$ and $x_1 = 1.1$. That is

$$\frac{f(1.1) - f(1)}{1.1 - 1} = \frac{1.21 - 1}{1.1 - 1} = 2.1.$$

More precisely Solution: If you want to make the answer more precisely, you can choose $x_1 = 1.001$. Then the slope of the tangent line at $x_0 = 1$ can be estimated by the slope to the secant line between $x_0 = 1$ and $x_1 = 1.001$. That is

$$\frac{f(1.001) - f(1)}{1.1 - 1} = \frac{1.001^2 - 1}{1.1 - 1} = 2.001.$$

In fact, the **precise** answer for the slope of the tangent line at $x_0 = 1$ equals 2. We will learn this latter.



Example 2. in the last class. The following data shows a company spending on marketing in these years. Show work and give **units** for each answer.

	1	2	3	4	5	6	7	8
year	2008	2009	2010	2011	2012	2013	2014	
Spending(million dollars)	23.07	24.47	26.21	30.36	38.31	46.38	57.96	62.800

(a). Let x stand for the number of years since 2007, and let $g(x)$ stand for the money spending on market in millions. Fit **the Exponential model** to the data. Round all coefficients to 3 decimal places.

$y = a \cdot b^x$ million dollars

$a = 17.752 \quad b = 1.171$

(b). Use the model in part (a) to estimate the **change** of the spending on market in millions between 2010 and 2015.

$x_0 = 3 \quad x_1 = 8$

Change $= g(8) - g(3) = 34.289$ million dollars

(c). Use the model in part (a) to estimate the **percentage change** of the spending on market in millions between 2010 and 2015.

$x_0 = 3 \quad x_1 = 8$

$$\text{Percentage change} = \frac{g(x_1) - g(x_0)}{g(x_0)} \cdot 100\% = \frac{g(8) - g(3)}{g(3)} \cdot 100\% = 120.264\%$$

(d). Use the model in part (a) to estimate the **average rate of change** of the spending on market in millions between 2010 and 2015.

$$ARC = \frac{g(x_1) - g(x_0)}{x_1 - x_0} = \frac{g(8) - g(3)}{8 - 3} = 6.858$$

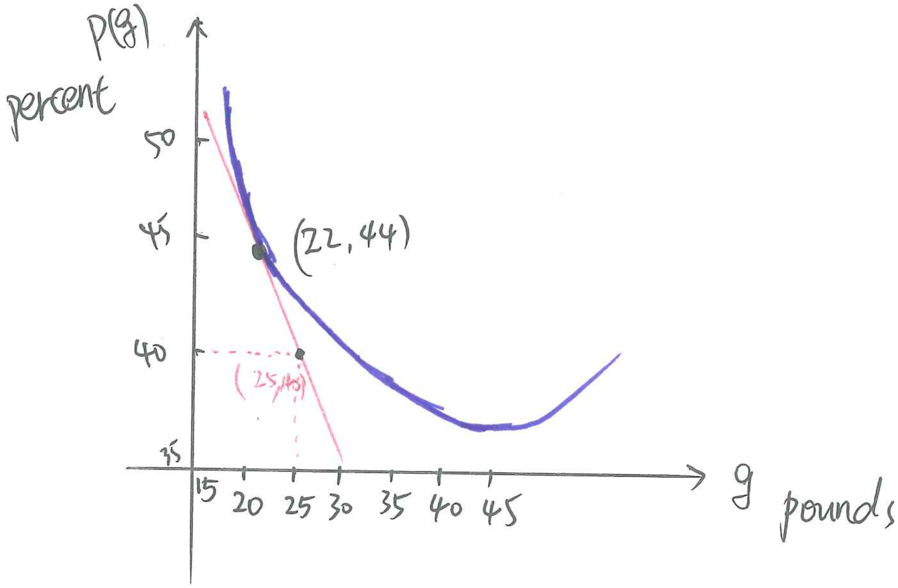
million dollars per year.

(3). Use the model in part (a) to estimate the (instantaneous) **rate of change** of the spending on market in millions in 2010. $X = 2010 - 2007 = 3$

$$\text{Rate of change} \approx \frac{g(3.001) - g(3)}{3.001 - 3} = 4.503$$

million dollars per year

Example (§2.2 HW12, page 149)



- Estimate a second point on the tangent line. (25, 40)
- $\text{Rate of change} \approx \frac{44-40}{22-25} = \frac{4}{-3} = -\frac{4}{3}$

percent per pound