Review Chain Rule

$$f(u(x))' = f'(u) \cdot u'(x)$$

Example: $f(x) = \ln(x^4 + 2)$

$u = x^4+2$

$f'(x) = f'(u) \cdot u'(x) = \dfrac{4x^3}{x^4+2}$

$f(x) + C = \int f'(x)dx = \int \dfrac{4x^3}{x^4+2} dx$

Integral by u-substitution

1. If $u(x)$ is a differentiable function, the **differential of** u **is** $du = u'(x)dx$.

Practice in ClassPacket 1-6.

2. Integration by u-substitution.

Practice in ClassPacket 7-19.

Example: Compute $\displaystyle\int \frac{5x^3}{x^4+2}\,dx$

Step 1

- $u = x^4+2$
- $du = 4x^3dx \Rightarrow dx = \frac{1}{4x^3} du$

Step 2 $\displaystyle\int \frac{5x^3}{x^4+2} dx = \int \frac{5\cancel{x^3}}{u} \cdot \frac{1}{4\cancel{x^3}} du = \frac{5}{4}\ln|u| + C$

$= \frac{5}{4}\ln(x^4+2) + C$

MATH 1231 **Integration by Substitution Problems**

Calculate the differential, du, of each function.

1. $u = 4x + 1$ $du = 4\,dx$
2. $u = x^4 + 2$ $du = 4x^3\,dx$
3. $u = 3x^2 + 1$ $du = 6x\,dx$
4. $u = \ln(x)$ $du = \frac{1}{x}\,dx$
5. $u = 1/x$ $du = -x^{-2}\,dx$
6. $u = e^x + 2$ $du = e^x\,dx$

Evaluate the given integral, i.e., find the indicated general antiderivative. In each case show u and du. Note that problem 19 is a little different from the others and illustrates the power of the method of substitution.

7. $\int (4x + 1)^4\,dx$

$u = 4x+1$

$du = 4\,dx \Rightarrow dx = \frac{du}{4}$

$\int (4x+1)^4 dx = \int \frac{u^4}{4}\,du = \frac{u^5}{4\times 5} + C = \frac{(4x+1)^5}{20} + C$

8. $\int e^{-2x+3}\,dx$

$u = -2x+3$

$du = -2dx \Rightarrow dx = -\frac{1}{2}du$

$\int e^{-2x+3} dx = \int e^u (-\frac{1}{2}) du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-2x+3} + C$

9. $\int \frac{5x^3}{x^4 + 2}\,dx$

$u = x^4+2$

$du = 4x^3 dx \Rightarrow dx = \frac{1}{4x^3}du$

$\int \frac{5x^3}{x^4+2} dx = \int \frac{5x^3}{u}\frac{1}{4x^3} du = \frac{5}{4}\ln|u| + C = \frac{5}{4}\ln(x^4+2) + C$

10. $\int 7x(3x^2 + 1)^5\,dx$

$u = 3x^2+1$

$du = 6x\,dx \Rightarrow dx = \frac{1}{6x}du$

$\int 7x(3x^2+5)^5 dx = \int 7x u^5 \frac{1}{6x} du = \frac{7}{36}u^6 + C = \frac{7}{36}(3x^2+1)^6 + C$

11. $\int \frac{4}{5x + 2}\,dx$

$u = 5x+2$

$du = 5dx \Rightarrow dx = \frac{1}{5}du$

$\int \frac{4}{5x+2} dx = \int \frac{4}{u}\cdot\frac{1}{5} du = \frac{4}{5}\ln|u| + C = \frac{4}{5}\ln(5x+2) + C$

12. $\int \frac{2}{(3x + 2)^4}\,dx$

$u = 3x+2$

$du = 3\,dx \Rightarrow dx = \frac{1}{3}du$

$\int \frac{2}{(3x+2)^4} dx = \int \frac{2}{u^4}\frac{1}{3} du = \int \frac{2}{3}u^{-4} du$

$= -2u^{-3} + C = -2(3x+2)^{-3} + C$

13. $\int x\sqrt{x^2+5}\,dx$ $= \int x u^{\frac{1}{2}} \frac{1}{2x}\,du = \int \frac{1}{2} u^{\frac{1}{2}}\,du = \frac{1}{3} u^{\frac{3}{2}} + C = \frac{1}{3}(x^2+5)^{\frac{3}{2}} + C$

$u = x^2+5$

$du = 2x\,dx \Rightarrow dx = \frac{1}{2x}\,du$

14. $\int \frac{1}{x\ln(x)}\,dx$ $= \int \frac{1}{x\cdot u} \cdot x \cdot du = \int \frac{1}{u}\,du = \ln u + C = \ln(\ln x) + C$

$u = \ln x$

$du = \frac{1}{x}dx \Rightarrow dx = x\,du$

15. $\int e^{x^2}\,x\,dx$ $= \int e^u \cdot x \cdot \frac{1}{2x}\,du = \int \frac{1}{2} e^u\,du = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$

$u = x^2$

$du = 2x\,dx \Rightarrow dx = \frac{1}{2x}\,du$

16. $\int 5^{3x+4}\,dx$ $= \int 5^u \cancel{\frac{1}{3}}\,du = \frac{1}{3}\cdot\frac{5^u}{\ln 5} + C = \frac{5^{3x+4}}{3\ln 5} + C$

$u = 3x+4$

$du = 3\,dx \Rightarrow dx = \frac{1}{3}\,du$

17. $\int \frac{2^{1/x}}{x^2}\,dx$ $= \int \frac{2^u}{x^2}(-x^2)\,du = \int -2^u\,du = -\frac{2^u}{\ln 2} + C = -\frac{2^{\frac{1}{x}}}{\ln 2} + C$

$u = \frac{1}{x}$

$du = -x^{-2}\,dx \Rightarrow dx = -x^2\,du$

18. $\int \frac{e^x}{e^x+2}\,dx$ $= \int \frac{e^x}{u} \cdot \frac{1}{e^x}\,du = \int \frac{1}{u}\,du = \ln u + C = \ln(e^x+2) + C$

$u = e^x+2$

$du = e^x\,dx \Rightarrow dx = \frac{1}{e^x}\,du$

19. $\int x\sqrt{x+1}\,dx$

$u = x+1 \Rightarrow x = u-1$

$du = dx$

$$\int x\sqrt{x+1}\,dx = \int (u-1)u^{\frac{1}{2}}\,du = \int u^{\frac{3}{2}} - u^{\frac{1}{2}}\,du = \frac{u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$$