The Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a)$$

where $F(x)$ is an antiderivative of $f(x)$, that is $F'(x) = f(x)$.

2. Revised from Textbook Page382 HW16. Corporate Revenue A corporation's revenue flow rate can be modeled as

$$r(x) = -3x^2 + 38x + 56 \text{ million dollars per year}$$

where x is the number of years since 1980.

a. Evaluate $\int_0^5 r(x)dx$.

b. Interpret the answer from part a.

c. From 1980 to 2000, in which year did the company has the maximal revenue?

(a) $\int_0^5 r(x)dx = -x^3+19x^2+56x\Big|_0^5$

$= -5^3+19\cdot 5^2+56\times 5 - (0)$

$= 630$ million dollars

1. MATH / fnInt(

$\int_{\boxed{0}}^{\boxed{5}} \boxed{Y1}\, d\boxed{X} = 630$

(b) The corporation's revenue increased by 630 million dollars between 1980 and 1985

(c) Solve $r(x)=0$

$-3x^2+38x+56=0$

$\begin{matrix} -1 & & 14 \\ & \times & \\ 3 & & 4 \end{matrix}$

$(-x+14)(3x+4)=0$

$\underline{x=14}$ $x=-\frac{4}{3}$

In 1994, the company has maximal revenue

(c) Solve $r(x)=0$ by Calculator

$Y1 = -3x^2+38x+56$

Windows: Xmin = 0

Xmax = 20

ZoomFit.

2nd / Calc / Zero $\Rightarrow x=14$

So In 1994 the company has maximal revenue.

1. Revised from Textbook page368. Based on market research, a toy manufacturer expects (from experience and market research) that marginal revenues from the sale of a new character in its line of 6-in. vinyl ponies will follow the pattern in the table. Total revenue from the sales of 50,000 ponies is expected to be $155,000. $R(50)=155$

Table 5.18 Marginal Revenue (to the producer) from the sale of vinyl toy ponies

Quantity (thousand ponies)	5	20	35	50
Marginal Revenue (dollars per pony)	19.56	10.62	5.76	3.12

a. Write an exponential model for marginal revenue $M(q)$ in terms of q thousand ponies.
b. Write a model for the total expected revenue $R(q)$ from the sale of q thousand ponies.
c. Evaluate $\int_6^{25} M(q)dq$.
d. Interpret the answer from part c.
~~e.~~ If the manufacturer wants to maximize the revenue, how many ponies should be sold?

(a.) $M(q) = 24(0.96^q)$ dollars per pony

(b) $R(q) = \int M(q)\,dq = \int 24(0.96^q)\,dq = 24\dfrac{0.96^q}{\ln(0.96)} + C$ thousand dollars

$R(50) = 24\dfrac{0.96^{50}}{\ln(0.96)} + C = 155$

$C = 155 - \dfrac{24(0.96^{50})}{\ln(0.96)} \approx 231$

Check by $\boxed{\text{MATH}/\text{fn Int}}$

(c) $\int_6^{25} 24(0.96^q)\,dq = \dfrac{24(0.96^q)}{\ln(0.96)}\Big|_6^{25} = 248.314$ thousand dollars

(d) When production is increased from 6 thousand to 25 thousand ponies, the Revenue increases by 248.314 thousand dollars.

~~(e)~~ Solve $M(q) = 0$

$24(0.96^q) = 0$

No solution

3. Textbook Page381 HW14. Phone Calls The rate of change of the number of international telephone calls billed in the United States between 1980 and 2000 can be described by

$$p(x) = 32.432e^{0.1826x} \text{ million calls per year}$$

where x is the number of years since 1980.

a. Evaluate $\int_5^{15} p(x)dx$.

b. Interpret the answer from part a.

(a) $\int_5^{15} 32.432e^{0.1826x}dx = \frac{32.432e^{0.1826x}}{0.1826}\Big|_5^{15} \approx 2305.357$ million calls

(b) Between 1985 and 1995, the number of International calls billed in the United States increased by 2305.4 million calls

Homework: 4. Textbook Page383 HW24. DVD Marginal Cost

(a) $C'(x) = (7.714\times10^{-5})x^2 - 0.047x + 8.940$ dollars per CD

(b) $C(x) = \int C'(x)dx = (2.571\times10^{-5})x^3 - 0.024x^2 + 8.94x + K$

Solve $C(150) = 750$ get $K \approx -$ ~~143.87~~ 137.77

(c) $\int_{200}^{300} C'(x)\,dx = C(x)\Big|_{200}^{300} \approx 207.55$ by fnInt

≈ 182.49 by $C(300) - C(200)$

When production increase from 200 to 300 CD per hour, the hourly cost increase 207.55 \$