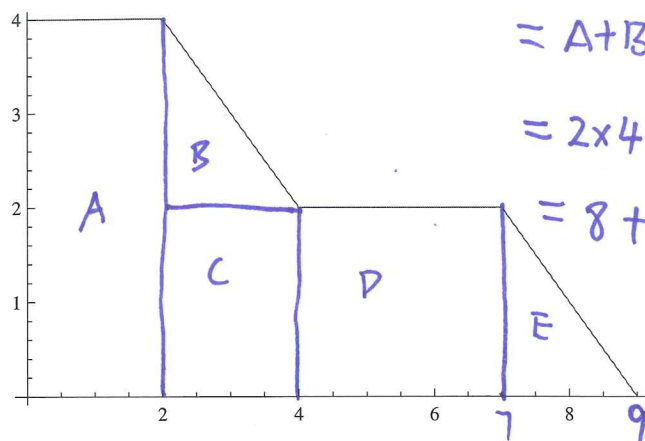
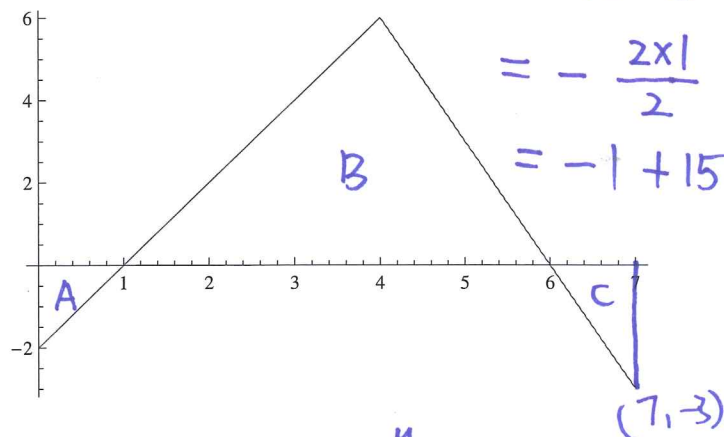


1. The graph of the function  $f(x)$  below consists of a series of line segments through the the following points in order:  $(0, 4)$ ,  $(2, 4)$ ,  $(4, 2)$ ,  $(7, 2)$  and  $(9, 0)$ . Compute  $\int_0^9 f(x) dx$ .



$$\begin{aligned}
 &= A+B+C+D+E \\
 &= 2 \times 4 + \frac{2 \times 2}{2} + 2 \times 2 + 3 \times 2 + \frac{2 \times 2}{2} \\
 &= 8 + 2 + 4 + 6 + 2 = 22
 \end{aligned}$$

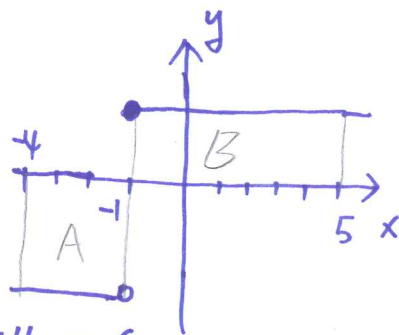
2. The graph of the function  $g(x)$  below consists of a line segment between  $(0, -2)$  and  $(4, 6)$  and a line segment between  $(4, 6)$  and  $(7, -3)$ . Compute  $\int_0^7 g(x) dx$ .



$$\begin{aligned}
 &= -A+B-C \\
 &= -\frac{2 \times 1}{2} + \frac{5 \times 6}{2} - \frac{1 \times 3}{2} \\
 &= -1 + 15 - 1.5 = 12.5
 \end{aligned}$$

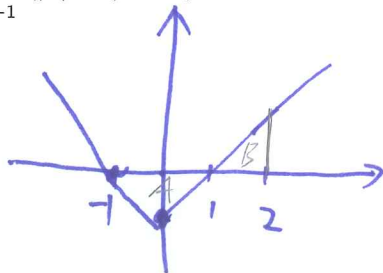
3. Let  $h(x) = \begin{cases} -6 & \text{if } x < -1, \\ 4 & \text{if } x \geq -1. \end{cases}$

Sketch  $h(x)$  and find  $\int_{-4}^5 h(x) dx$ .



$$\int_{-4}^5 h(x) dx = -A+B = -3 \times 6 + 6 \times 4 = 6$$

4. Find  $\int_{-1}^2 (|x| - 1) dx$ . (Hint: Sketch the graph of the function:  $j(x) = |x| - 1$ .)



$$= \begin{cases} x-1 & x \geq 0 \\ -x-1 & x < 0 \end{cases}$$

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$$\int_{-1}^2 (|x| - 1) dx = -A+B = \frac{-2 \times 1}{2} + \frac{1 \times 1}{2} = -0.5$$

MATH 1231 Additional Problems on Definite Integrals

In problems 1–8, evaluate the given integral. Give numerical answers to three decimal places. The answer to problem 7 may be left in terms of  $e$ . Show all work. Use your calculator only to perform basic arithmetic.

$$1. \int_0^3 6x \, dx = 3x^2 \Big|_0^3 = 27 - 0 = 27$$

$$2. \int_{-2}^0 (12x^2 + 1) \, dx = 4x^3 + x \Big|_{-2}^0 = 0 - (4(-2)^3 - 2) = 34$$

$$3. \int_1^4 \frac{1}{2\sqrt{x}} \, dx = \int_1^4 \frac{1}{2} x^{-\frac{1}{2}} \, dx = x^{\frac{1}{2}} \Big|_1^4 = 2 - 1 = 1$$

$$4. \int_1^2 \left( 5t^{-3} + \frac{2}{t} \right) \, dt = \frac{5t^{-2}}{-2} + 2 \ln t \Big|_1^2 = \frac{15}{8} + 2 \ln 2 \approx 3.261$$

$$5. \int_4^9 \sqrt{x^3} \, dx = \frac{2}{5} x^{\frac{5}{2}} \Big|_4^9 = 84.4$$

$$6. \int_0^2 10 \left( \frac{1}{3} \right)^x \, dx = \frac{10 \cdot \left( \frac{1}{3} \right)^x}{\ln \left( \frac{1}{3} \right)} \Big|_0^2 = \frac{10 \left( \frac{1}{3} \right)^2}{\ln \left( \frac{1}{3} \right)} - \frac{10}{\ln \left( \frac{1}{3} \right)} \approx 8.091$$

$$7. \int_0^1 e^{-0.2t} \, dt = \frac{e^{-0.2t}}{-0.2} \Big|_0^1 = \frac{e^{-0.2}}{-0.2} + 5$$

$$8. \int_2^5 \frac{d(\ln(x+1))}{dx} \, dx = \ln(x+1) \Big|_2^5 = \ln 6 - \ln 3 \approx 0.693$$

$$9. \text{Find } \frac{d}{dx} \left( \int_0^x 2^t \, dt \right) = 2^x$$

10. Suppose that  $f(x)$  and  $g(x)$  are two polynomial functions such that  $f(x) = g'(x)$ ,  $f(2) = 9$ , and  $g(2) = 14$ .

If  $\int_2^5 f(x) \, dx = 36$ , find the value of  $g(5)$ .

$g(x)$  is an antiderivative of  $f(x)$

$$\int_2^5 f(x) \, dx = g(x) \Big|_2^5 = g(5) - g(2) = 36$$

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$$g(5) = 36 + g(2) = 36 + 14 = 50$$