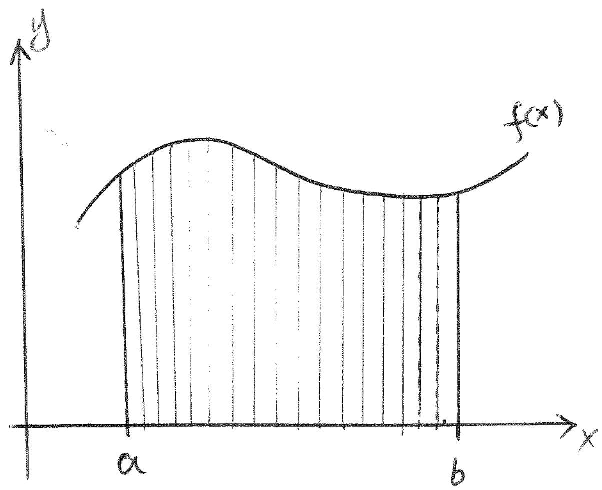
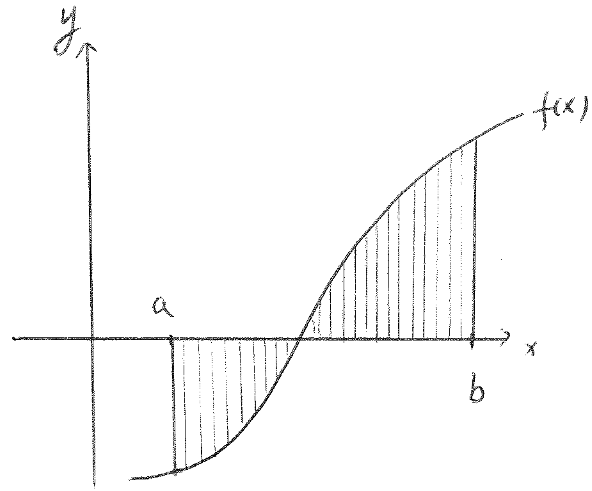
1. Area under a curve.





2. Definite Integral

Let $f(x)$ be a continuous function defined on the interval $[a, b]$. The **definite integral** (accumulated change) of $f(x)$ from a to b is

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x.$$

3. The relation between area under a curve and the definite Integral

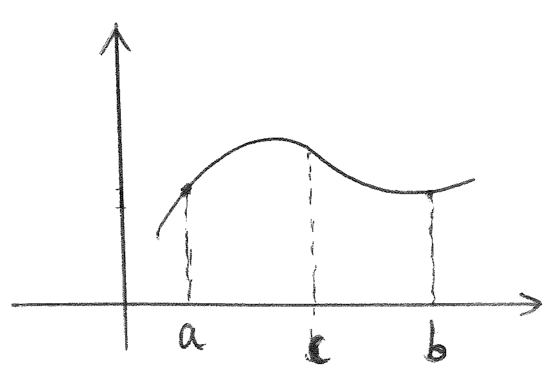
$$\int_a^b f(x)dx = \text{(The Area above x-axis)} - \text{(The Area under x-axis)}$$

4. More properties definite integral

$$\int_a^b f(x) \pm g(x)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

$$\int_a^b kf(x)dx = k\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

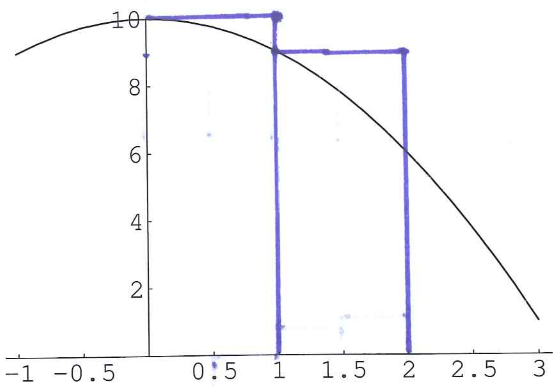


MATH 1231 Worksheet on Area Approximation with Rectangles

1a. Use 2 left rectangles to approximate the area under the curve $f(x) = 10 - x^2$ and above the interval [0, 2]. Sketch the rectangles using the graph of $f(x)$ below, and find L_2, the sum of their areas. L_2 = 19

$\Delta x = \frac{2-0}{2} = 1$

x	$f(x)$
0	10
1	9

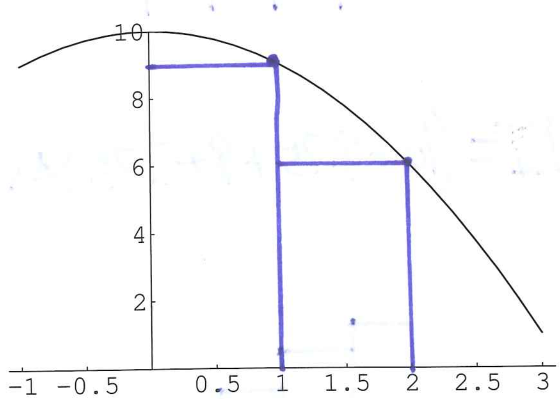


$L_2 = (10+9)\Delta x = 19$

1b. Use 2 right rectangles to approximate the area under the curve $f(x) = 10 - x^2$ and above the interval [0, 2]. Sketch the rectangles using the graph of $f(x)$ below, and find R_2, the sum of their areas. R_2 = 15

$\Delta x = \frac{2-0}{2} = 1$

x	$f(x)$
1	9
2	6



$R_2 = (9+6)\Delta x = 15$

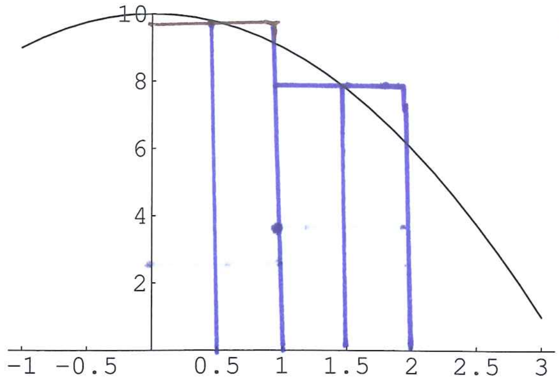
1c. Use 2 midpoint rectangles to approximate the same area as in (1a) and (1b). Sketch the rectangles using the graph of $f(x)$ below, and find M_2, the sum of their areas. M_2= 17.5 .

$\Delta x = \frac{2-0}{2} = 1$

$M_2 = (9.75 + 7.75)\Delta x$

$= 17.5$

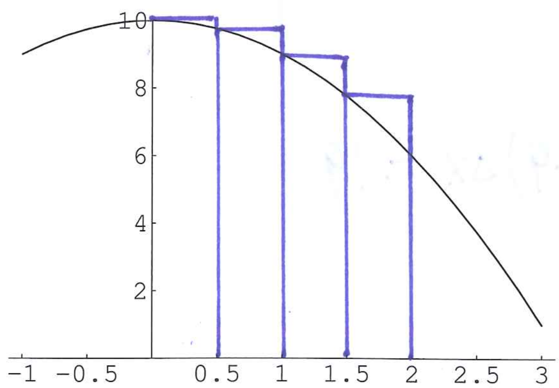
x	$f(x)$
0.5	9.75
1.5	7.75



2a. Use 4 left rectangles to approximate the same area as in problem 1. Find the sum L_4. $L_4 =$ 18.25.

$\Delta x = \frac{2-0}{4} = 0.5$

x	$f(x)$
0	10
0.5	9.75
1	9
1.5	7.75

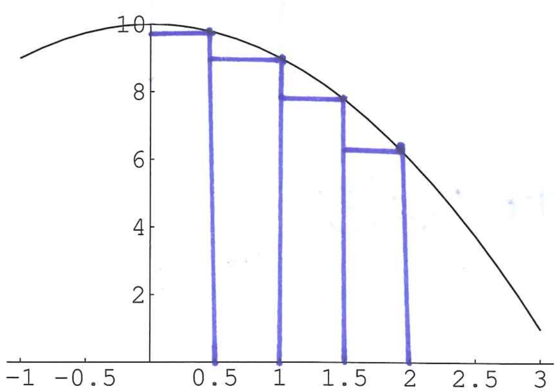


$L_4 = (10 + 9.75 + 9 + 7.75)\Delta x = 18.25$

2b. Use 4 right rectangles to approximate the same area as in problem 1. Find the sum R_4. $R_4 =$ 16.25.

$\Delta x = \frac{2-0}{4} = 0.5$

x	$f(x)$
0.5	9.75
1	9
1.5	7.75
2	6

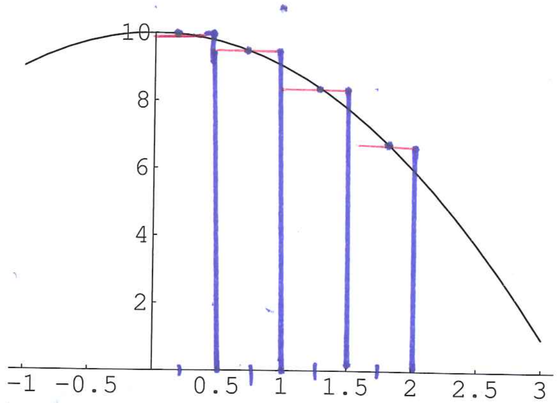


$R_4 = (9.75 + 9 + 7.75 + 6)\,0.5 = 16.25$

2c. Use 4 midpoint rectangles to approximate the same area as in problem 1. Show that $M_4 = 17.375$ square units. Then find the value of the following quantity: $(1/3)\left(\frac{L_4 + R_4}{2}\right) + (2/3)M_4 =$ 17.33. This quantity is a very accurate approximation of the area.

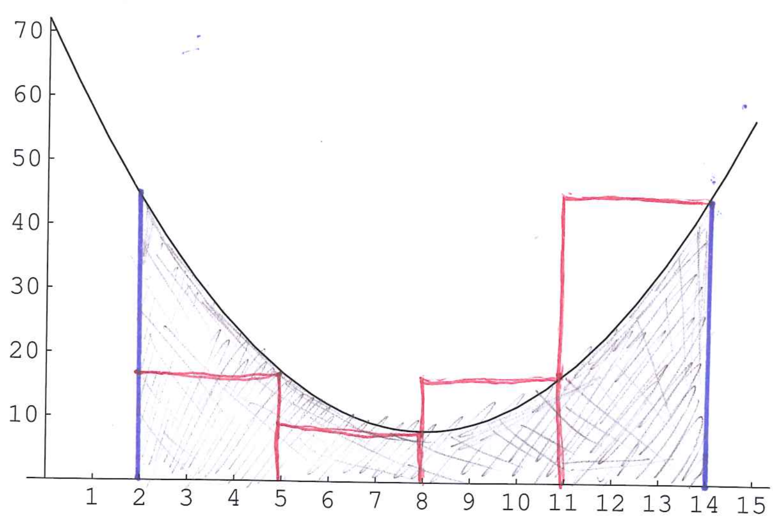
$\Delta x = \frac{2-0}{4} = 0.5$

x	$f(x)$
0.25	9.9375
0.75	9.4375
1.25	8.4375
1.75	6.9375



$M_4 = (9.9375 + 9.4375 + 8.4375 + 6.9375)\,0.5 = 17.375$

3a. A portion of the graph of $y = x^2 - 16x + 72$ is given below. Make a careful sketch of the region whose area is given by the definite integral: $\int_2^{14} (x^2 - 16x + 72)\, dx$. **Then shade the region.**



$\Delta x = \frac{14-2}{4} = 3$

Red color for 3b

3b. Estimate the area (in square units) of the region in part (a) using the **4 right rectangle approximation.**

[2, 14]

x	$f(x)$
5	17
8	8
11	17
14	44

$R_4 = (17 + 8 + 17 + 44)\Delta x$

$= 86 \times 3$

$= 258$

MATH 1231: Additional Anti-derivative Problems

In problems 1–5, find an antiderivative of the given function.

1. $g(x) = 6x^2 - 4x^6$ $\quad G(x) = 2x^3 - \frac{4}{7}x^7$

2. $f(x) = 3e^x - \frac{1}{x} + 12$ $\quad F(x) = 3e^x - \ln|x| + 12x$

3. $h(x) = 3^x - x^3$ $\quad H(x) = \frac{3^x}{\ln 3} - \frac{x^4}{4}$

4. $m(x) = 12x^{-3} + 6\sqrt{x} - 100$ $\quad M(x) = -6x^{-2} + 4x^{\frac{3}{2}} - 100x$

5. $r(x) = 13(1.05)^x - x$ $\quad R(x) = 13\frac{(1.05^x)}{\ln 1.05} - \frac{x^2}{2}$

In problems 6–11, evaluate the given integral, i.e., find the indicated general antiderivative.

6. $\int x(x+1)\, dx$ $= \int x^2 + x\, dx = \frac{x^3}{3} + \frac{x^2}{2} + C$

7. $\int \left(\frac{x+2}{x}\right) dx$ $= \int 1 + \frac{2}{x}\, dx = x + 2\ln|x| + C$

8. $\int (x+1)^3\, dx$ $= \frac{(x+1)^4}{4} + C$

9. $\int \frac{d(\ln(x+1))}{dx}\, dx$ $= \ln(x+1) + C$

10. $\frac{d}{dx}\left(\int \ln(x+1)\, dx\right)$ $= \ln(x+1)$

11. $\int \left(\frac{1}{2\sqrt[3]{x^4}} + \sqrt{x^3}\right) dx$ $= \int \frac{1}{2}x^{-\frac{4}{3}} + x^{\frac{3}{2}}\, dx = -\frac{3}{2}x^{-\frac{1}{3}} + \frac{2}{5}x^{\frac{5}{2}} + C$

12. $\int \frac{1}{2x+1}\, dx$ $= \frac{\ln|2x+1|}{2} + C$