1. The General and Specific Antiderivative

• The **general antiderivative** (indefinite integral) of $f(x)$ is

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is an antiderivative and C is an arbitrary constant number.
• When the constant C is known, $F(x) + C$ is a **specific antiderivative**.

$$\int k dx = kx + C \qquad \int x^{-1} dx = \ln|x| + C \qquad \int e^{kx} dx = \frac{e^{kx}}{k} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int b^x dx = \frac{b^x}{\ln b} + C \qquad \int e^x dx = e^x + C$$

2. Finding a specific anti-derivative

Example: Write a formula for F, the specific antiderivative of f.

1. $f(x) = 6x^2 + 16; \quad F(2) = 37.$

$\int f(x)\,dx = \frac{6x^3}{3} + 16x + C$

$= 2x^3 + 16x + C$

$F(2) = 2\cdot 2^3 + 16\cdot 2 + C = 37$

$48 + C = 37$

$C = -11$

$F(x) = 2x^3 + 16x - 11$

2. $f(u) = \frac{2}{u} + u; \quad F(1) = 5$

$\int f(u)\,du = 2\ln|u| + \frac{u^2}{2} + C$

$F(1) = 2\underbrace{\ln(1)}_{=0} + \frac{1}{2} + C = 5$

$C = 4.5$

$F(u) = 2\ln|u| + \frac{u^2}{2} + 4.5$

3. $f(x) = 3e^{2x} + 15x^5; \quad F(0) = 8$

$\int f(x)\,dx = \frac{3e^{2x}}{2} + \frac{15x^6}{6} + C$

$F(0) = \frac{3}{2} + C = 8$

$C = 6.5$

$F(x) = \frac{3e^{2x}}{2} + \frac{5x^6}{2} + 6.5$

Example: (HW23 in Textbook Page 364)
Fuel Consumption. The rate of change of the average annual fuel consumption of passenger vehicles, buses, and trucks from 1970 through 2000 can be modeled as

$$f(t) = 0.8t - 15.9 \text{ gallons per vehicle per year}$$

where t is the number of years since 1970. The average annual fuel consumption was 712 gallons per vehicle in 1980. (Source: Based on data from Bureau of Transportation Statistics)

$t=10$ $F(10)=712$

Q: Write the specific antiderivative giving the average annual fuel consumption.

$F(t)=?$ $\int f(t)\,dt = \frac{0.8t^2}{2} - 15.9t + C$

$F(10) = \frac{0.8\times 100}{2} - 159 + C = 712$

$C = 831$

$F(t) = 0.4t^2 - 15.9t + 831$ gallons/vehicle.

Example: (HW21 in Textbook Page 373)
Investment Growth An investment worth \$1 million in 2005 has been growing at a rate of where t is the number of years since 2005. $f(t) = 0.140(1.15^t)$ million \$ per year

Q: Calculate how much the investment will have grown between 2005 and 2015 and how much it is projected to grow between 2015 and 2020.

$F(t) = \int f(t)\,dt = \int 0.140(1.15^t)\,dt$

$= \frac{0.140(1.15^t)}{\ln 1.15} + C$

- $F(10) - F(0) \approx 3.051$ million \$
- $F(15) - F(10) \approx 4.098$ million \$.

Quiz6Review

$F(0)=1$ $\frac{0.14}{\ln 1.15} + C = 1$ million \$

$C \approx -0.0017$ $F(t) = 1.0017(1.15)^t - 0.0017$