

1. The Antiderivative

- Let f be a function of x . A function F is called an **antiderivative** of f if

$$\frac{d}{dx}[F(x)] = f(x)$$

The derivative formula

$f(x)$	$f'(x)$
b	0
x^n	$n \cdot x^{n-1}$
b^x	$(\ln b) \cdot b^x$
e^{kx}	$k \cdot e^{kx}$
e^x	e^x
$\ln(x)$	$\frac{1}{x}$
$\ln(kx)$	$\frac{1}{x}$

The Anti-derivative formula

$f(x)$	$F(x)$
k	kx
x^n ($n \neq -1$)	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln x $
b^x	$\frac{b^x}{\ln b}$
e^{kx}	$\frac{e^{kx}}{k}$
e^x	e^x

Example: Compute the anti-derivative of the following functions

1. $f(x) = 4x^2 + 2$

$$F(x) = \frac{4x^3}{3} + 2x$$

2. $f(x) = \frac{5}{x} + 2x^{-4} + 3$

$$F(x) = 5 \ln|x| + \frac{2x^{-3}}{-3} + 3x$$

3. $g(x) = 3^x + 5e^{2x}$

$$G(x) = \frac{3^x}{\ln 3} + \frac{5e^{2x}}{2}$$

4. $g(x) = 10x^{-2} - 3\sqrt[4]{x^3} - 3e^x$

$$G(x) = \frac{10x^{-1}}{-1} - \frac{3 \cdot X^{\frac{3}{4}+1}}{\frac{3}{4}+1} - 3e^x$$

2. The General and Specific Antiderivative

- The **general antiderivative** (indefinite integral) of $f(x)$ is

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is an antiderivative and C is an arbitrary constant number.

- When the constant C is known, $F(x) + C$ is a **specific antiderivative**.

$\int k dx = kx + C$	$\int x^{-1} dx = \ln x + C$	$\int e^{kx} dx = \frac{e^{kx}}{k} + C$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\int b^x dx = \frac{b^x}{\ln b} + C$	$\int e^x dx = e^x + C$

Example: Compute the general anti-derivative of the following functions

1. $f(x) = \frac{2}{x} + 2x^{-3} + 1$

2. $g(x) = 12(1.3^x) - 3x^{1.2} - 3e^x$

$$\int f(x) dx = 2 \ln|x| + \frac{2x^{-2}}{-2} + x + C$$

$$\int g(x) dx = \frac{12 \cdot 1.3^x}{\ln 1.3} - \frac{3x^{2.2}}{2.2} - 3e^x + C$$

3. Connection between Derivatives and Integrals

For a differentiable function $f(x)$, we have the following formulas

- $\frac{d}{dx} \int f(x) dx = f(x)$

- $\int \frac{df}{dx} dx = f(x) + C.$

Example: Compute the following functions

1. $\int \frac{d(x^3(\ln(x^3 + e^x)))}{dx} dx = x^3 \ln(x^3 + e^x) + C$

2. $\frac{d}{dx} \left(\int \frac{x^7 + xe^x}{\ln(x^4 + \ln x)} dx \right) = \frac{x^7 + xe^x}{\ln(x^4 + \ln x)}$