- **Critical points.**

> A **critical point** of a continuous function $f(x)$ is a point $(c, f(c))$ at which $f(x)$ is not differentiable, or $f'(c) = 0$.

1. Definition Test for Relative Extrema

Suppose c is a critical input. Compute the value of $f(c)$, $f(c-h)$ and $f(c+h)$ for a small number h. (For example, h can be 0.1, or 1, or ...). Then use the definition.

2. The First Derivative Test for Relative Extrema

Suppose c is a critical input of a continuous function $f(x)$.
- If $f'(x)$ changes from positive(+) to negative(−) at c, then $f(c)$ is relative maximum.
- If $f'(x)$ changes from negative(−) to positive(+) at c, then $f(c)$ is relative minimum.
- If $f'(x)$ does not change sign at c then $f(c)$ is not a relative extreme point.

3. The Second Derivative Test

> Suppose the function $f(x)$ is a function such that $f'(c) = 0$.
> - If $f''(c) > 0$, then $f(c)$ is a relative minimum.
> - If $f''(c) < 0$, then $f(c)$ is a relative maximum.

- If $f''(c) = 0$, we could **NOT** make a conclusion by this method.

See Example in Lec17 again. $f(x)=3x^3-1.5x^2-20x$

X **Example:** Find all the critical points for function $f(x) = 2x^3 - 3.5x^2 + 2x - 3$. Show all steps especially factoring. write down both x and y coordinates for the critical points. Using three different method to determine the relative extreme point.

$f'(x)=6x^2-7x+2=0$

$$\begin{matrix} 2 & & -1 \\ & \times & \\ 3 & & -2 \end{matrix}$$

$(2x-1)(3x-2)=0$

$2x-1=0 \quad 3x-2=0$

$x=\frac{1}{2} \quad x=\frac{2}{3}$

①

x	0	$\frac{1}{2}$	0.6	$\frac{2}{3}$	1
$f(x)$	-3	-2.625	-2.628	-2.630	-2.5

relative maximum

relative minimum

②

x	0	$\frac{1}{2}$	0.6	$\frac{2}{3}$	1
$f'(x)$	2	0	-0.04	0	1

+ − − +

$f''(x)=12x-7$

③

x	$\frac{1}{2}$	$\frac{2}{3}$
$f''(x)$	-1	1

concave down concave up

$\frac{1}{2}$ relative maximum $\frac{2}{3}$ relative minimum

Ex: $f(x) = 3x^3 - 1.5x^2 - 20x$

Q1: Find all critical points.

$f'(x) = 0$

$9x^2 - 3x - 20 = 0$

$\begin{matrix} 3 & & 4 \\ & X & \\ 3 & & -5 \end{matrix}$

$(3x+4)(3x-5) = 0$

$x_1 = -\frac{4}{3}$ ≈ -1.33 $x_2 = \frac{5}{3}$ ≈ 1.667

Q2: Using 3 different method for testing the extrema.

① Definition Test

x	-2	$-\frac{4}{3}$	-1
f(x)	10	16.89	15.5

So $x = -\frac{4}{3}$ is a relative maximum

x	1	$\frac{5}{3}$	2
f(x)	-18.5	-23.61	-22

So $x = \frac{5}{3}$ is a relative minimum.

② 1st derivative Test.

x	-2	-1
f(x)	22	-8

$+ \quad -$

So $x = -\frac{4}{3}$ is a relative maximum

x	1	2
f(x)	-14	10

$- \quad +$

So $x = \frac{5}{3}$ is a relative minimum

③ 2nd derivative Test

x	$-\frac{4}{3}$	$\frac{5}{3}$
$18x-3 = f''(x)$	-27	27

concave down   concave up

So $x = -\frac{4}{3}$ is a relative maximum

$x = \frac{5}{3}$ is a relative minimum

Q3: Find the inflection point for f(x).

$f''(x) = 18x - 3 = 0$

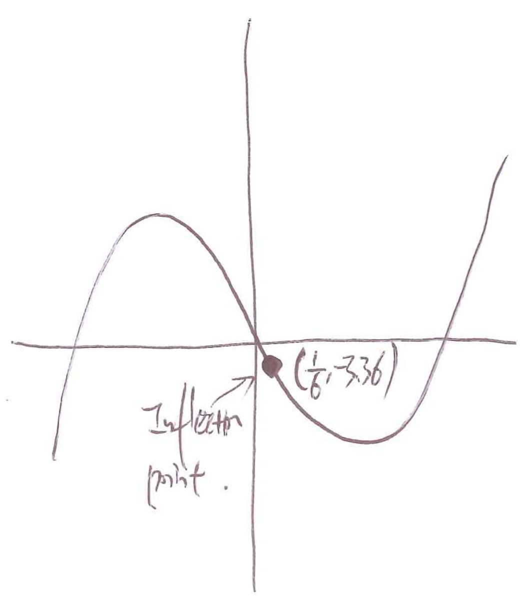
$x = \frac{1}{6}$

$f(\frac{1}{6}) = -3.36$

The inflection point is $(\frac{1}{6}, -3.36)$

Q4: Draw the graph of f(x) and lable the inflection point:

Answer for Q4:



• window:

$X_{min} = -4$

$X_{max} = 4$

$Y_{min} = -30$

$Y_{max} = 30$