1. Second Derivative.

- The second derivative is the derivative of the derivative function, denoted by $f''(x)$.
- The units can be seen in the following picture.(Picture 4.39 from textbook.)

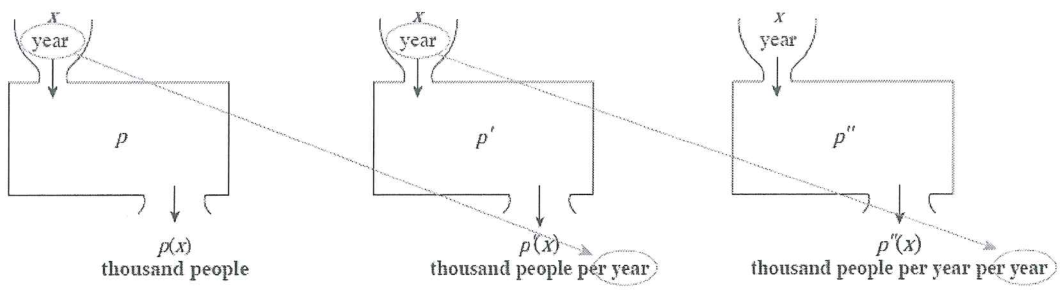


Figure 4.39

Example:1 Write the first and second derivatives of the function $f(x) = 2x^3 - 5.5x^2 - 7x$.

$$f'(x) = 6x^2 - 11x - 7$$

$$f''(x) = 12x - 11$$

4. The Second Derivative Test

> **The Second Derivative Test for Relative Extrema**
>
> Suppose the function $f(x)$ is a function such that $f'(c) = 0$.
> - If $f''(c) > 0$, then $f(c)$ is a relative minimum.
> - If $f''(c) < 0$, then $f(c)$ is a relative maximum.

- If $f''(c) = 0$, we could NOT make a conclusion by this method.

Analysis the following three functons

1. $f(x) = x^4$

$f'(x) = 4x^3$

$f''(x) = 12x^2$

$f''(0) = 0$

$x=0$ is minimum

x	-1	1
$f'(x)$	-4	4

2. $g(x) = x^3$

$g'(x) = 3x^2$

$g''(x) = 6x$

$g''(0) = 0$

Neither

x	-1	1
$f'(x)$	3	3

2. $h(x) = -x^4$

$h'(x) = -4x^3$

$h''(x) = -12x^3$

$h''(0) = 0$

$x=0$ is maximum.

x	-1	1
$f'(x)$	4	-4

Example: Find the critical points for function $f(x) = 2x^3 - 5.5x^2 - 7x$. Using the second derivative test to identify them as maximum or minimum.

$f'(x) = 0$

$6x^2 - 11x - 7 = 0$

$(2x+1)(3x-7) = 0$

$x = -\frac{1}{2}$ $\quad x = \frac{7}{3}$

$f''(x) = 12x - 11$

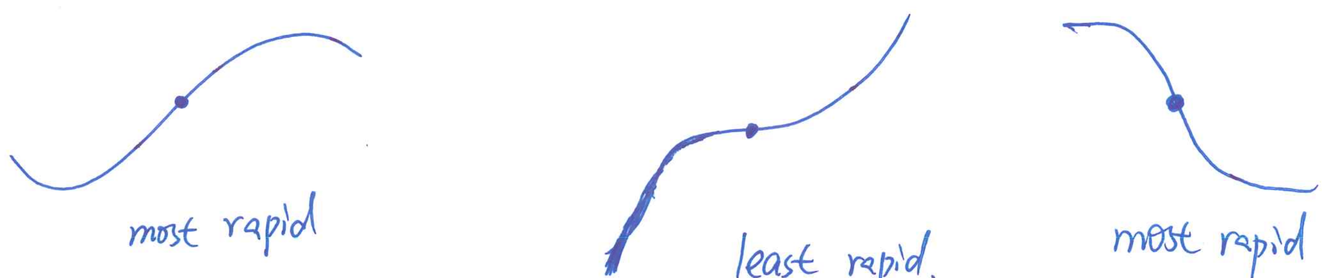
$f''(-\frac{1}{2}) = -17 < 0$ $\quad f''(\frac{7}{3}) = 17 > 0$

So $f(-\frac{1}{2})$ is a relative ~~minimum~~ maximum

$f(\frac{7}{3})$ is a relative ~~maximum~~ minimum

1. Inflection point

A point at which a graph changes concavity is an inflection point. It is a point of most rapid change or least rapid change.



3. The Second Derivative and Concavity

- $f''(x) > 0 \Leftrightarrow f'(x)$ is increasing $\Leftrightarrow f(x)$ is concave up
- $f''(x) < 0 \Leftrightarrow f'(x)$ is decreasing $\Leftrightarrow f(x)$ is concave down
- If $f''(c)$ changes signs, then c is a inflection point.

Summary: Let $f(x)$ be a differentiable function

Critical points = solutions of $f'(x) = 0$ $\xrightarrow{\text{Identify (3 methods)}}$ Relative minimum or Relative maximum or Neither

① Definition Test

② 1st derivative Test

③ 2nd derivative Test