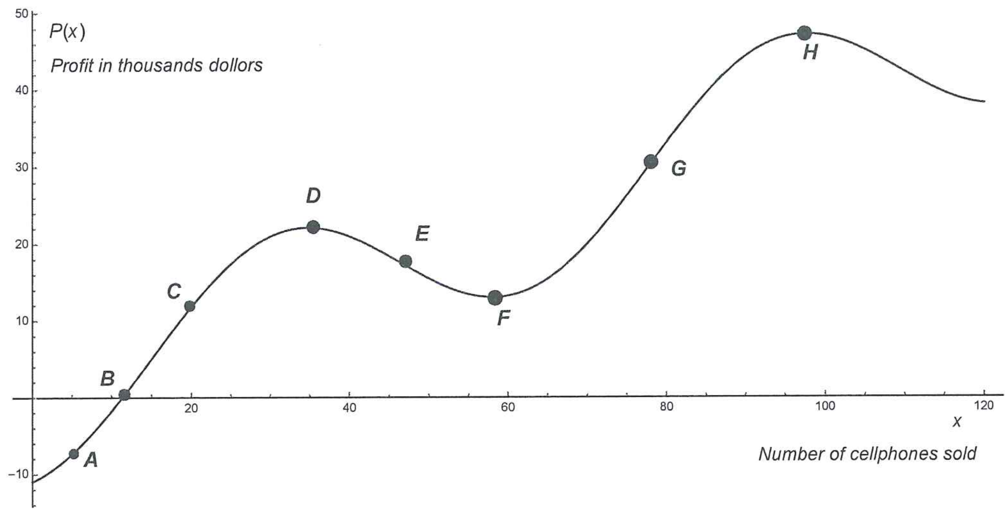
Relative Extrema: 1. Relative maximum 2. Relative minimum

Let $f(x)$ be a function.

♦ 1. The function $f(x)$ has a **relative maximum** at c if the output $f(c)$ is greater than any other output in some interval around c.

♦ 2. The function $f(x)$ has a **relative minimum** at c if the output $f(c)$ is smaller than any other output in some interval around c.

Example:



D and H are relative maxima and F is a relative minimum.

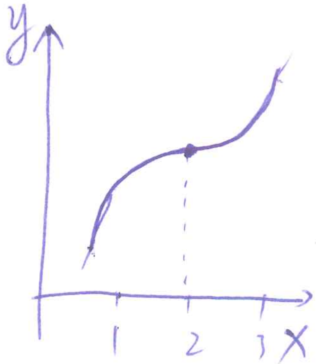
• **Absolute Exterme points** not required.

Critical points.

> A **critical point** of a continuous function $f(x)$ is a point $(c, f(c))$ at which $f(x)$ is not differentiable, or $\boxed{f'(c) = 0}$.

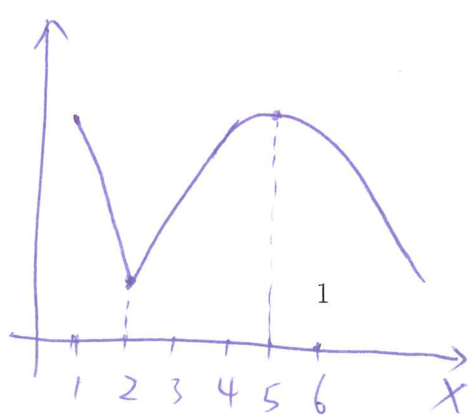
The input value c is called the critical input for the critical point $(c, f(c))$.

Example: Find the critical points, the maxima, the minima:



critical point

Not relative Extremum



critical points

Not critical points

1. Definition Test for Relative Extrema

Calculate 3 Value

Suppose c is a critical input. Compute the value of $f(c)$, $f(c-h)$ and $f(c+h)$ for a small number h. (For example, h can be 0.1, or 1, or ...). Then use the definition.

Example:1 Find all extreme points and indentify the extreme as a maximum or minimum for function $g(x) = -3x^2 + 14.1x - 16.2$

Solve $g'(x)=0$

$-6x+14.1=0$

$x=2.35$

x	2	2.35	3
$g(x)$	0	0.3675	-0.9

So $g(x)$ has a maximum at $x=2.35$

2. The First Derivative Test for Relative Extrema

Suppose c is a critical input of a continuous function $f(x)$.

- If $f'(x)$ changes from positive(+) to negative(−) at c, then $f(c)$ is relative maximum.
- If $f'(x)$ changes from negative(−) to positive(+) at c, then $f(c)$ is relative minimum.
- If $f'(x)$ does not change sign at c then $f(c)$ is not a relative extreme point.

Example: Using the first derivative test for Example 1.

x	2	3
$g'(x)$	2.1	-3.9

$g'(x)$ change from + to − at $x=2.35$

So, $g(2.35)$ is a maximum

Example: Find all extreme points and indentify the extreme as a maximum or minimum for the function $f(x) = x^3 + 4x^2 - 16x + 5$.

Window
$X_{min}=-5$
$X_{max}=5$
$Y_{min}=-10$
$Y_{max}=80$

Solve $f'(x)=0$

$3x^2+8x-16=0$

$\begin{matrix} 1 & & 4 \\ & \times & \\ 3 & & -4 \end{matrix}$

$(x+4)(3x-4)=0$

$x+4=0 \quad 3x-4=0$

$x=-4 \quad x=\frac{4}{3}$

Definition Test:

① $x=-4$

x	-5	-4	-3
$f(x)$	60	69	62

② $x=\frac{4}{3}\approx 1.333$

x	1	$\frac{4}{3}$	2
$f(x)$	-6	-6.85	-3

First Derivative test:

①

x	-5	-3
$f'(x)$	19	-13

So $f(-4)$ is a maximum

②

x	1	2
$f'(x)$	-5	12

$f(\frac{4}{3})$ is a minimum

Window

$X_{min} = -4$

$X_{max} = 4$

$Y_{min} = -30$

$Y_{max} = 30$

Example: $f(x) = 3x^3 - 1.5x^2 - 20x$

① Solve $f'(x) = 0$

$$9x^2 - 3x - 20 = 0$$

$$\begin{array}{ccc} 3 & & 4 \\ & \times & \\ 3 & & -5 \end{array}$$

$$(3x+4)(3x-5) = 0$$

$x = -\frac{4}{3}$ $\quad x = \frac{5}{3} \approx 1.667$

≈ -1.33

② Definition Test

① $x = -\frac{4}{3}$

x	-2	$-\frac{4}{3}$	-1
$f(x)$	10	16.89	15.5

② $x = \frac{5}{3}$

x	1	$\frac{5}{3}$	2
$f(x)$	-18.5	-23.61	-22

③ First Derive Test

① $x = -\frac{4}{3}$

x	-2	$\cancel{-\frac{4}{3}}$	-1
$f'(x)$	22	$\cancel{0}$	-8

$+ \quad -$

① $x = \frac{5}{3}$

x	1	$\cancel{\frac{5}{3}}$	2
$f'(x)$	-14	$\cancel{0}$	10

$- \quad +$

So

$f(-\frac{4}{3})$ is a maximum $\qquad f(\frac{5}{3})$ is a minimum