1. grading
2. WebAssign (10 times)

About Quiz4:

4. $\dfrac{2x^5 - 3^x}{2\sqrt[3]{x^4}}$. 6. x the number of year since 2000.

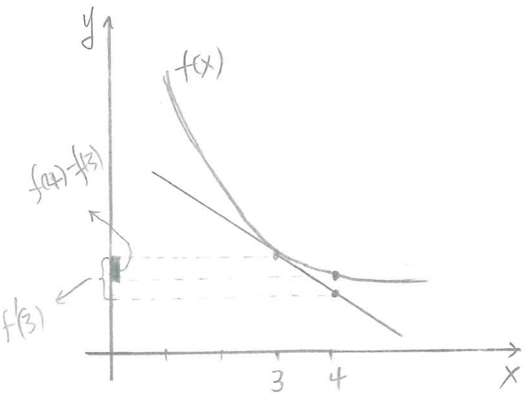
$= (2x^5 - 3^x)(\frac{1}{2}x^{-\frac{4}{3}})$

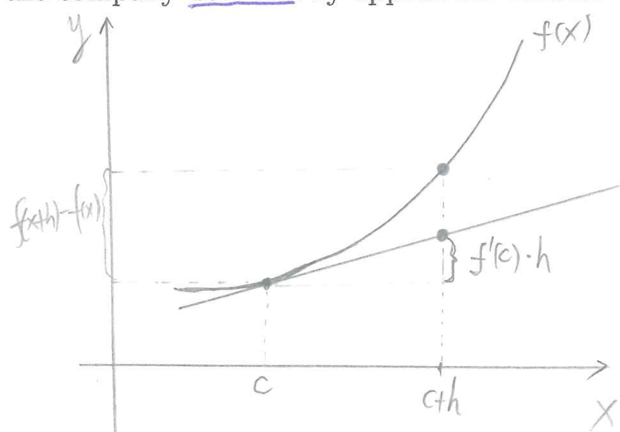
Year 1995 ($x = -5$)
2011 ($x = 11$)

1. Estimates the change
The known information: A point c and the rate of change(the derivative $f'(c)$).
The problem: Estimate the change $f(c+h) - f(c)$, where h is a small number.

Recall: Let $f(x)$ be the sale function of a company in million units, where x is number of years after 2001. What is the meaning of $f'(3) = -0.5$ million/year?

Answer: From 2004 to 2005, the sale of the company decrease by approx 0.5 million.





More Generally:

Approximate Change:	**Approximate the Result of change:**
$f'(c) \cdot h \approx f(c+h) - f(c)$	$f(c+h) \approx f(c) + f'(c) \cdot h$

Question1: In above example, estimate the change of the company's sale between 2004 and 2006.

$h = 5 - 3 = 2$ $\quad f(5) - f(3) \approx f'(3) \cdot 2 = -0.5 \times 2 = -1$ million.

Question2: If the sale in 2004 is 3.4 million units, estimate the the company's sale in 2006.

$f(3) = 3.4$ million $\quad f(5) = ?$

$f(5) \approx f(3) + f'(3) \cdot 2 = 3.4 - 1 = 2.4$ million

2. The Linearization

Using the tangent line of a function $f(x)$ at a point c to estimate the value of the function's value near c.

x^4 $4x^3$
e^x $\times$ e^x

Recall: (a) Find the **slope of the tangent line** at $x = 1$ of the function $f(x) = x^4 e^x$.

$f'(x) = x^4 e^x + 4e^x x^3$ $\qquad$ $f'(1) = e + 4e = 5e$

(b) Find a formula for the above tangent line.

$f(1) = e$ $\qquad$ $f(x) - f(1) = f'(1)\cdot(x-1)$ or $f(x) = e + 5e(x-1)$ or $f(x) = 5ex - 4e$

Linearization:

$$f_L(x) = f(c) + f'(c)\cdot(x-c)$$

Example We have the information $f(7) = 4$ and $f'(7) = -12.9$.

(a) Write a linearization for $f(x)$ with respect to x.

$f_L(x) = f(7) + f'(7)(x-7)$ $\qquad$ $f_L(x) = 4 + (-12.9)(x-7)$ or $f_L(x) = -12.9x + 94.3$

(b) Using the linearization to estimate $f(x)$ at $x = 7.25$.

$f(7.25) \approx f_L(7.25) = 0.775$

In the Project:

Unit price x, Unit cost c, Fixed cost C_0

Demanding function $D(x)$.

- Revenue function $R(x) = xD(x)$
- Cost function $C(x) = cD(x) + C_0$
- Profit function $P(x) = R(x) - C(x) = (x-c)D(x) - C_0$

Some times, we get the cost function $C(q)$ and the revenue function $R(q)$ with respect to the number of units q produced by the company.

- Cost function $C(q)$
- Revenue function $R(q)$
- Profit function $P(q) = R(q) - C(q)$

$\boxed{\blacktriangleright \textbf{ Marginal Cost} = C'(q)}$

$\boxed{\blacktriangleright \textbf{ Marginal Revenue} = R'(q)}$

Example 1. A inc. has found that the number of phones (in millions) it sells is given by the model: $D(x) = 15(0.971)^x$ where x is the selling price in dollars.
(a) Write down a model for $R(x)$ (dollars), the revenue, as a function of the price.

$R(x) = x \cdot D(x) = 15x(0.971)^x$ million \$

(b) Write down a formula for the rate of change of revenue as a function of price.

$R'(x) = 15(0.971)^x + 15x\ln(0.971)(0.971)^x$ million \$/\$

(c) When the price is \$300, what is the rate of change of revenue? Show work. Give your answer with units.

$R'(300) = -0.172$ million \$/\$

(d) If W inc. wants to maximize revenue, should it increase the price of sunglasses from \$300 to \$310? Explain the answer in a sentence **using your answer to part c)**.

No. Since the rate of change of revenue is Negative at 300\$
the Revenue is decreasing when the price is \$300

Example 2. A company makes T-shirts at a daily cost of $C(x) = 0.2x^3 - 0.6x^2 + 6.7x + 125.5$ dollars where x is the number of hundreds of T-shirts produced.
(a) Find the marginal cost function.

$C'(x) = 0.6x^2 - 1.2x + 6.7$ \$/hundred T-shires

(b) Find the marginal cost of producing 100 T-shirts. Show work. Give your answer with units.

$x=1$

$C'(1) = 6.1$ \$/hundred T-shires

(c) Assume that it costs \$131.8 to produce 100 T-shirts. Use this information and your answer to part (b) to **estimate** the cost of producing 110 T-shirts. Show work. Give your answer with units.

$h = 1.1 - 1 = 0.1$ $\quad C(1.1) \approx C(1) + C'(1)(0.1) = 131.8 + 6.1(0.1) = 132.41$ \$

Acurate $C(1.1) = 132.4102$