Math1231 Lecture 13

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Example1

A chain of music stores sells CDs. The demand, in hundreds of CDs, is modelled by the function:

$$D(x) = 56.6(0.93)^x$$

where x is the price of a CD in dollars.
(a) Find the function for the rate of change of demand function.

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$D'(x) = 56.6 \ln(0.93) 0.93^x$ hundreds CDs per dollar.

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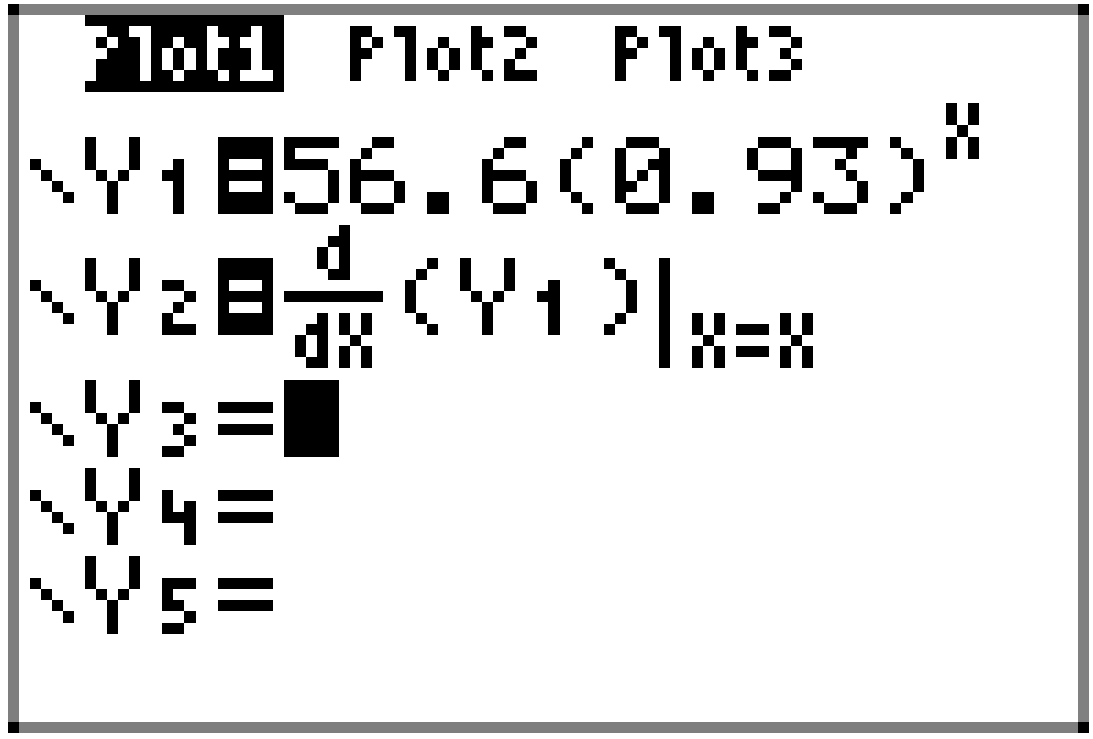
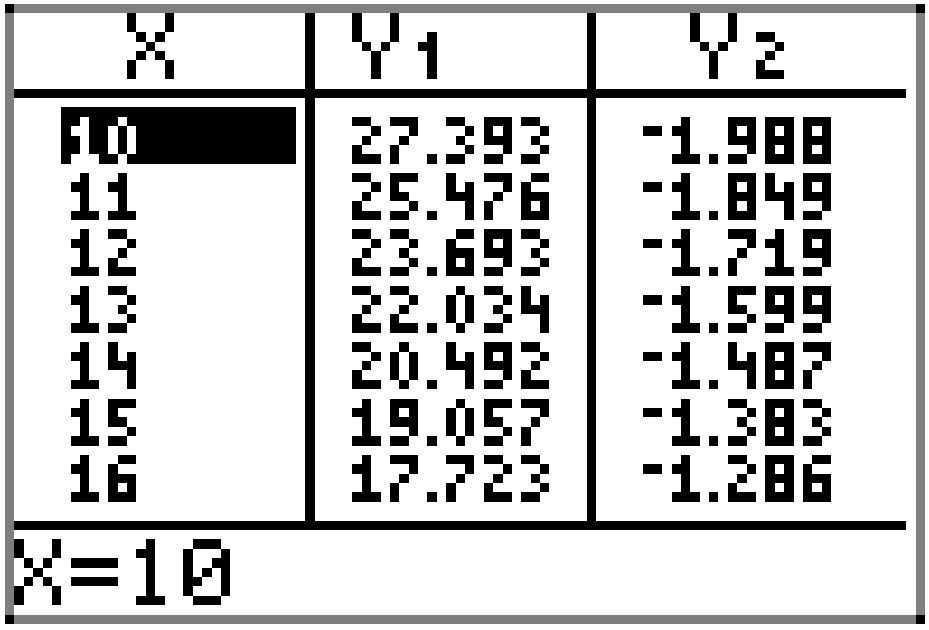
where x is the price of a CD in dollars.
(a) Find the function for the rate of change of demand function.
$D'(x) = 56.6\ln(0.93)0.93^x$ hundreds CDs per dollar.
(b) Fill in the following table. Round numerical results in the table to three decimal places.

				Units
x	10	15	20	
Demand				
Rate of change of Demand				


Plot1 Plot2 Plot3
\Y1=56.6(0.93)^X
\Y2=d/dX(Y1)|X=X
\Y3=
\Y4=
\Y5=


X
Y1
Y2
10
27.393
-1.988
11
25.476
-1.849
12
23.693
-1.719
13
22.034
-1.599
14
20.492
-1.487
15
19.057
-1.383
16
17.723
-1.286
X=10

X	Y1	Y2
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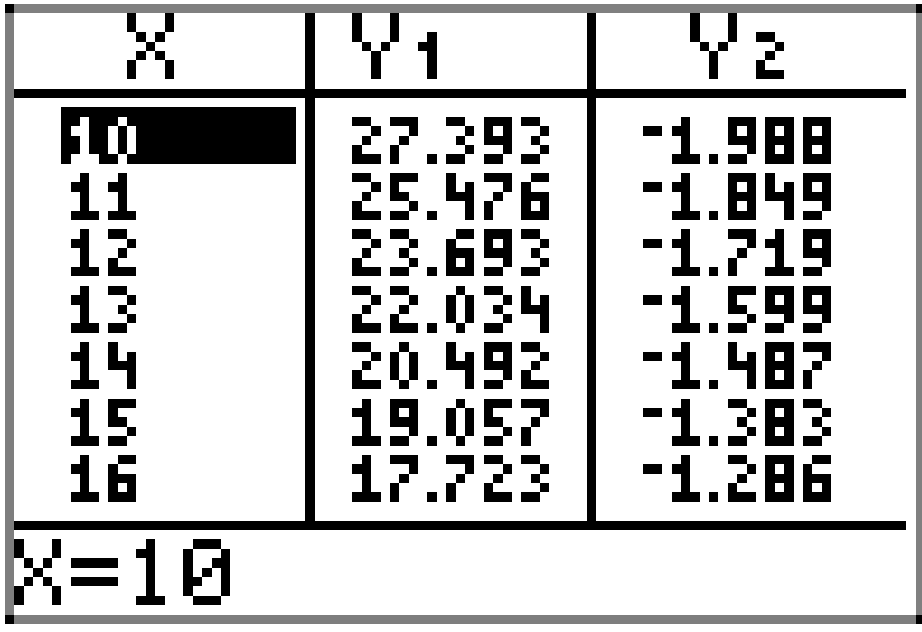
X	Y1	Y2
14	20.492	-1.487
15	19.057	-1.383
16	17.723	-1.286
17	16.483	-1.196
18	15.329	-1.112
19	14.256	-1.035
20	13.258	-.9621

X=14



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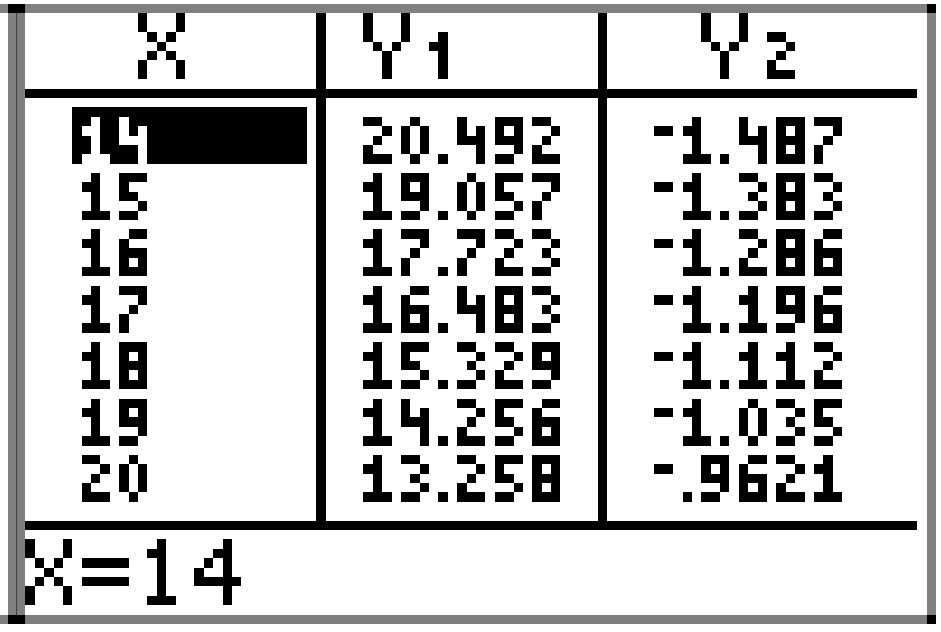
X=10



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				Units
x	10	15	20	dollar
Demand	27.393	19.057	13.258	hundreds CDs
Rate of change of Demand	-1.988	-1.383	-0.962	hundreds/dollar

Example2(Similar as Textbook 3.4 hw34)

Example(Similar as Textbook 3.4 hw34) The tuition x years since 1990 at a University is modeled to be

$$T(x) = 25012e^{0.054x} \text{ dollars}$$

(a) Write the rate of change formula for tuition.

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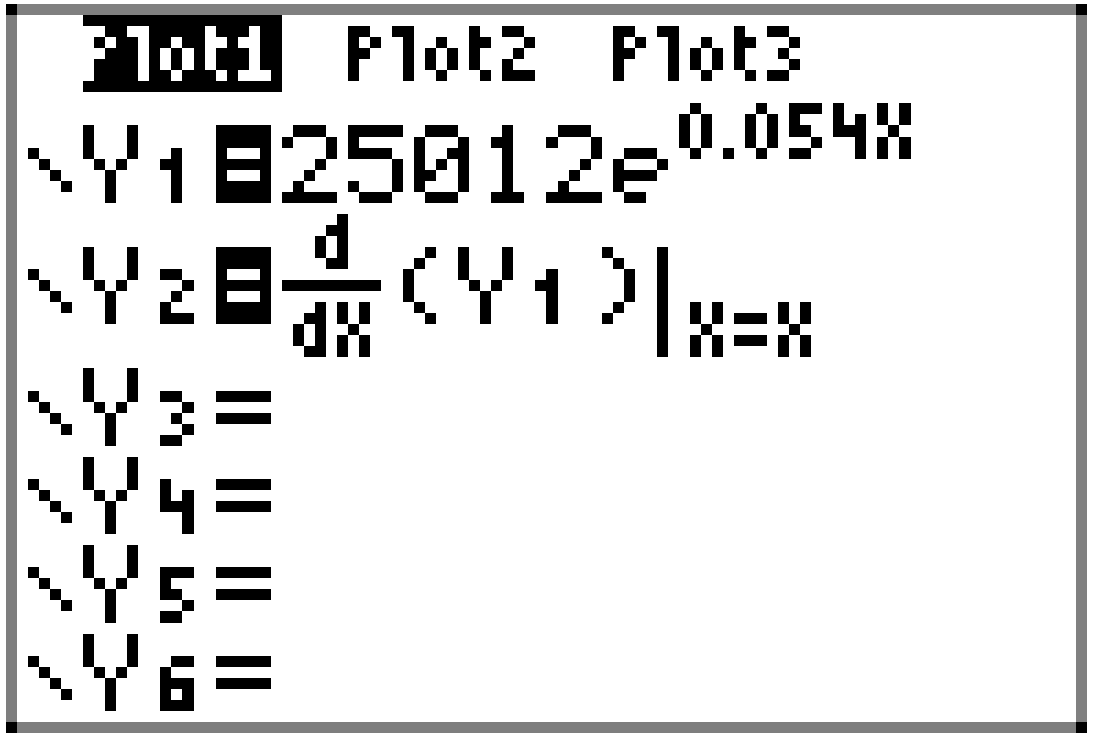
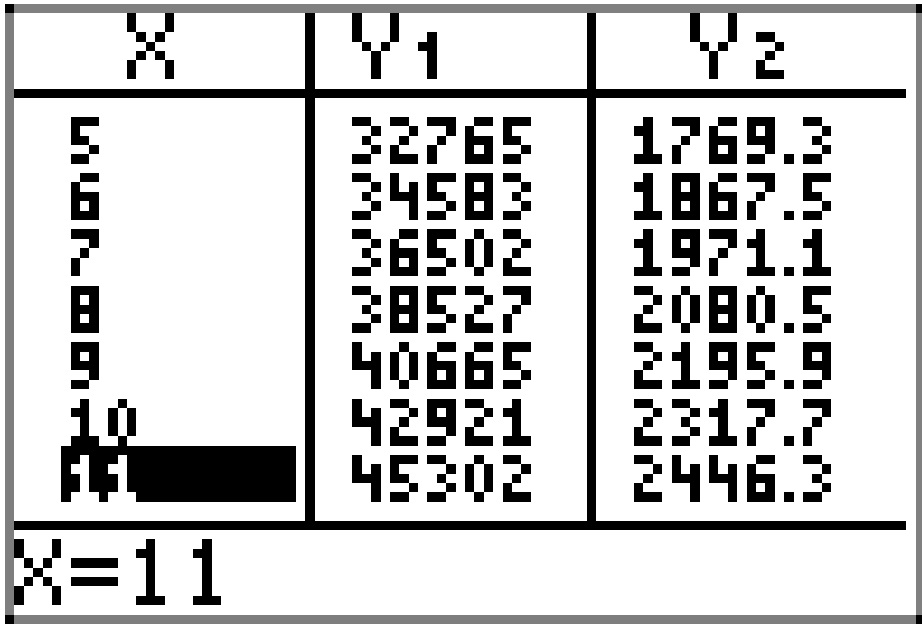
$$T(x) = 25012e^{0.054x} \text{ dollars}$$

(a) Write the rate of change formula for tuition.
$T'(x) = 25012(0.054)e^{0.054x}$ dollars/year
(b) Fill in the following table. Round numerical results in the table to three decimal places.

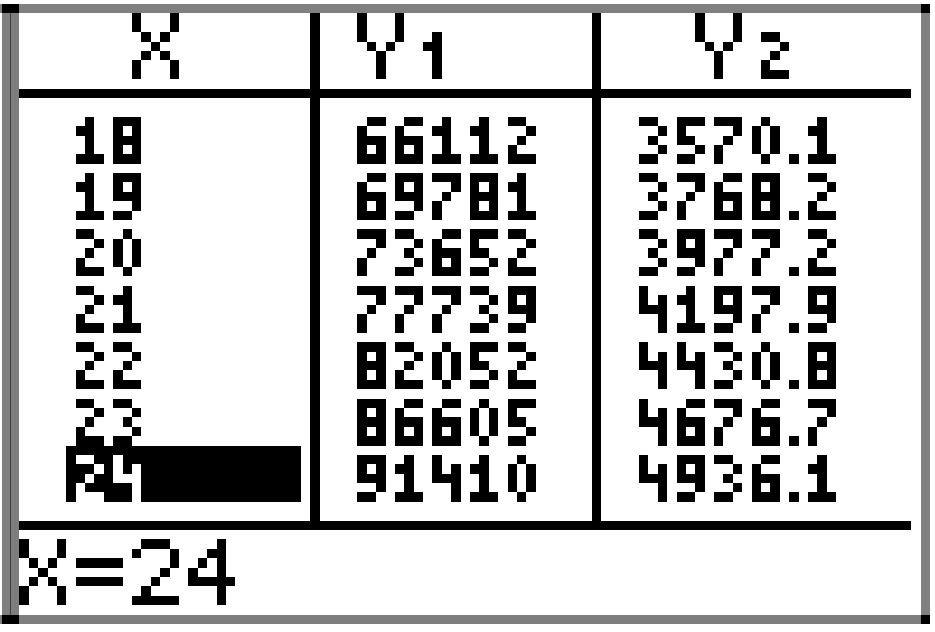
	1995	2000	2014	Units
x				
Tuition				
Rate of change of Tuition				


Plot1 Plot2 Plot3
\Y1 = 25012e^0.054X
\Y2 = d/dX (Y1)|X=X
\Y3 =
\Y4 =
\Y5 =
\Y6 =



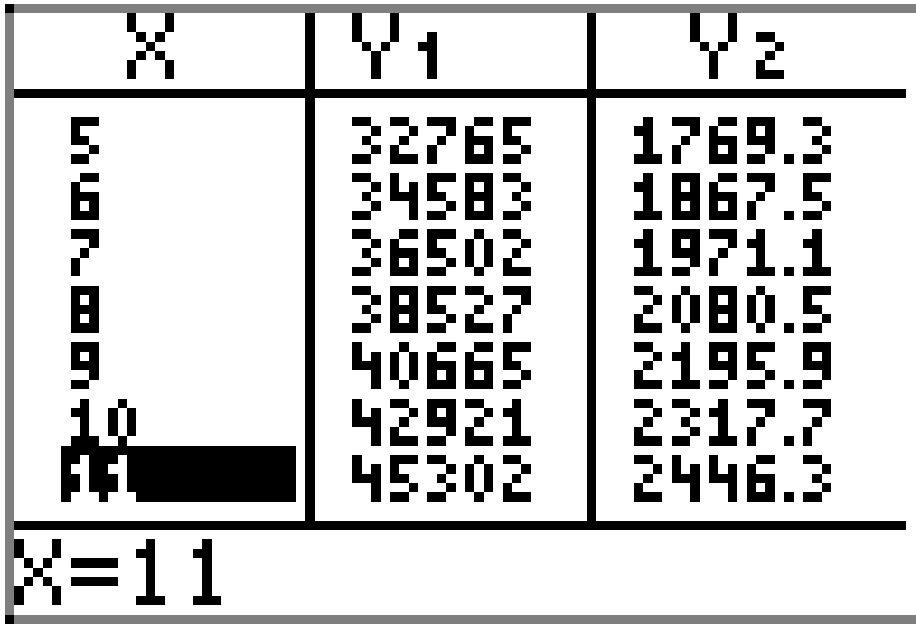
X	Y1	Y2
5	32765	1769.3
6	34583	1867.5
7	36502	1971.1
8	38527	2080.5
9	40665	2195.9
10	42921	2317.7
[redacted]	45302	2446.3

X=11



X	Y1	Y2
18	66112	3570.1
19	69781	3768.2
20	73652	3977.2
21	77739	4197.9
22	82052	4430.8
23	86605	4676.7
[redacted]	91410	4936.1

X=24



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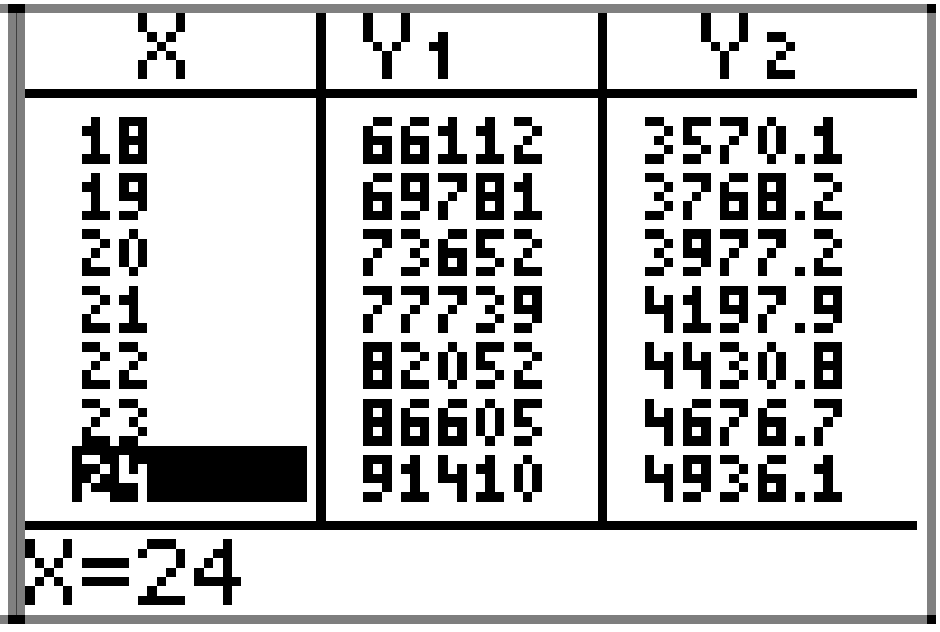

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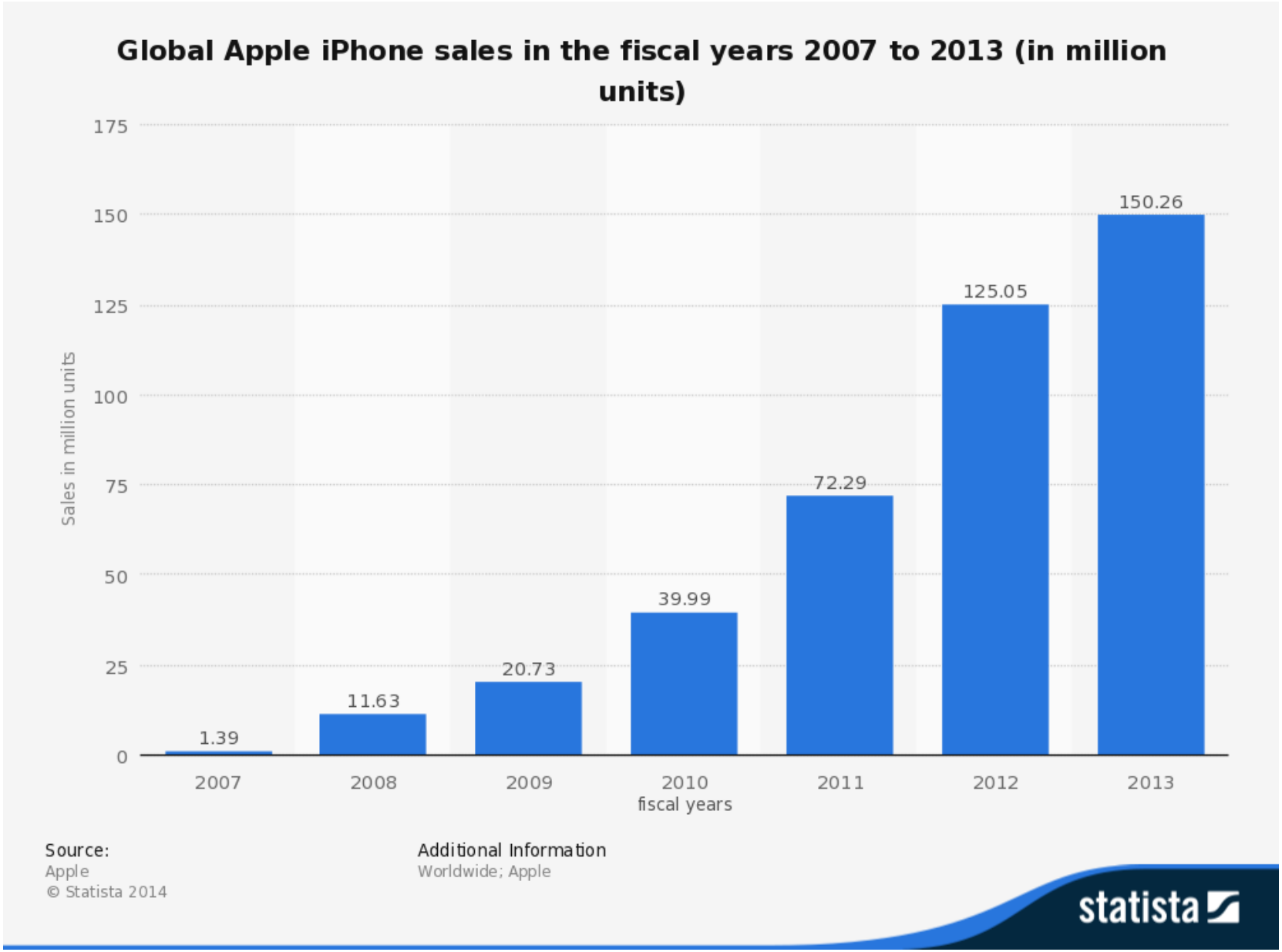
	1995	2000	2014	Units
x	5	10	24	year
Tuition	32765	42921	91410	$
Rate of change of Tuition	1769.3	2317.7	4936.1	$/year

Example3(Similar as Problem36 in HW 3.1.)



http://www.statista.com/statistics/276306/global-apple-iphone-sales-since-fiscal-year-2007/

# years since 2006	1	2	3	4	5	6	7
sales in million units	1.39	11.63	20.73	39.99	72.29	125.05	150.26

(a). Let x be the years since 2006, and let $S(x)$ be the sales of iPhones in million units. Fit the best model to the data.

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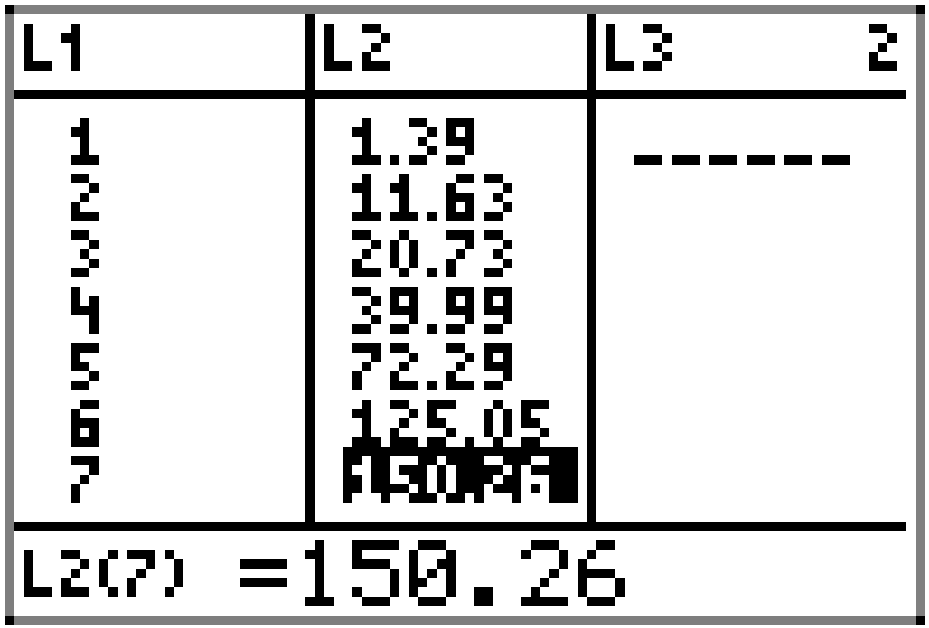

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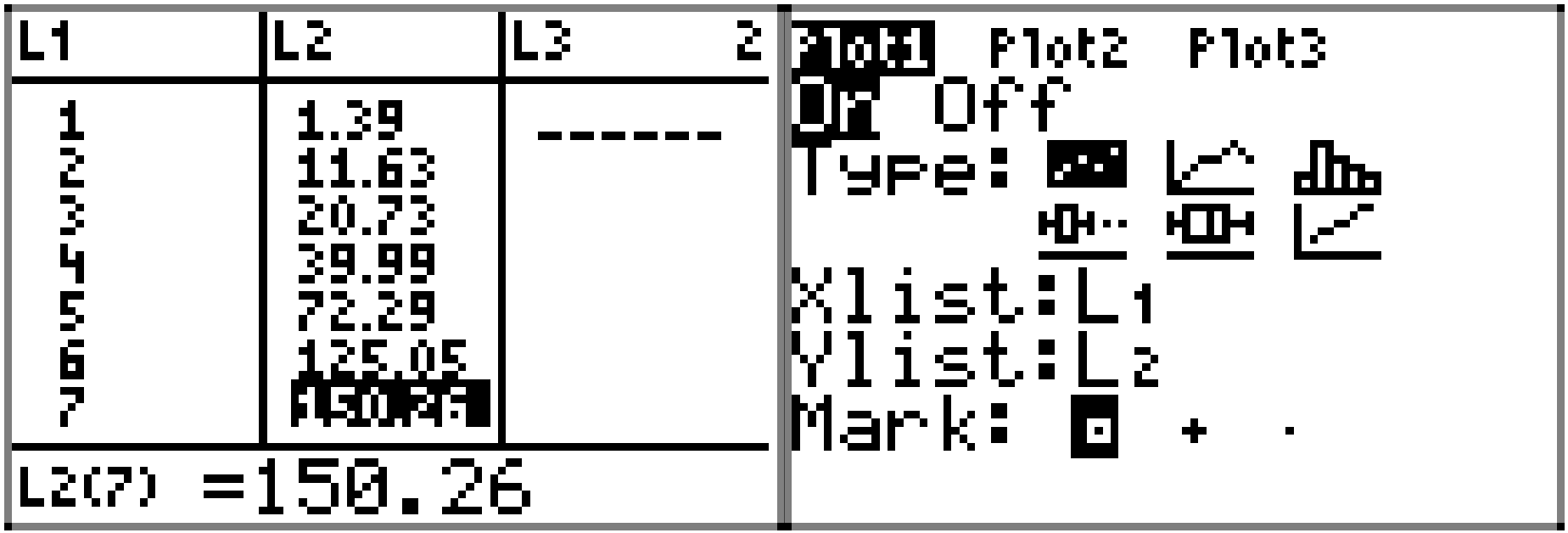
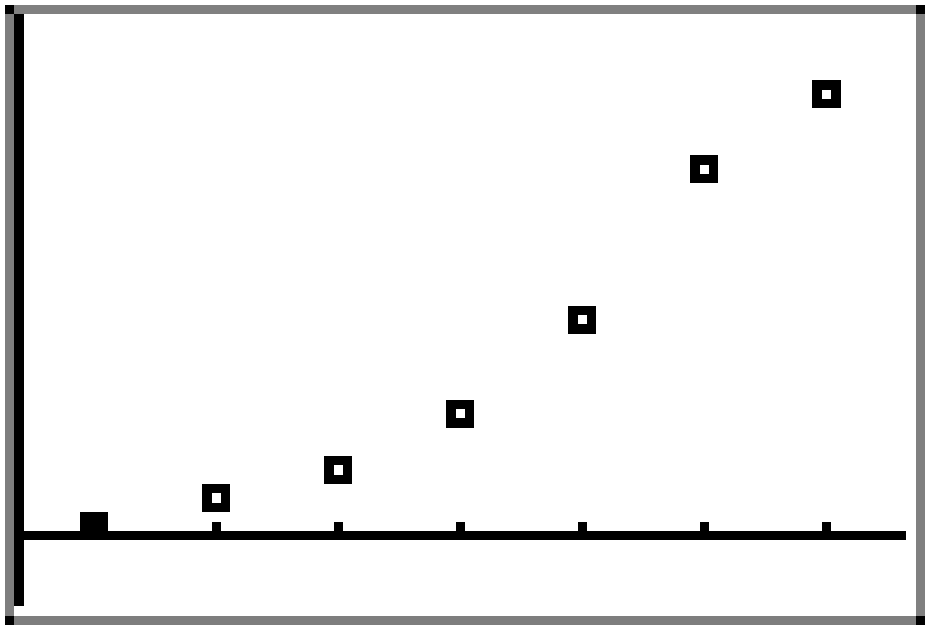
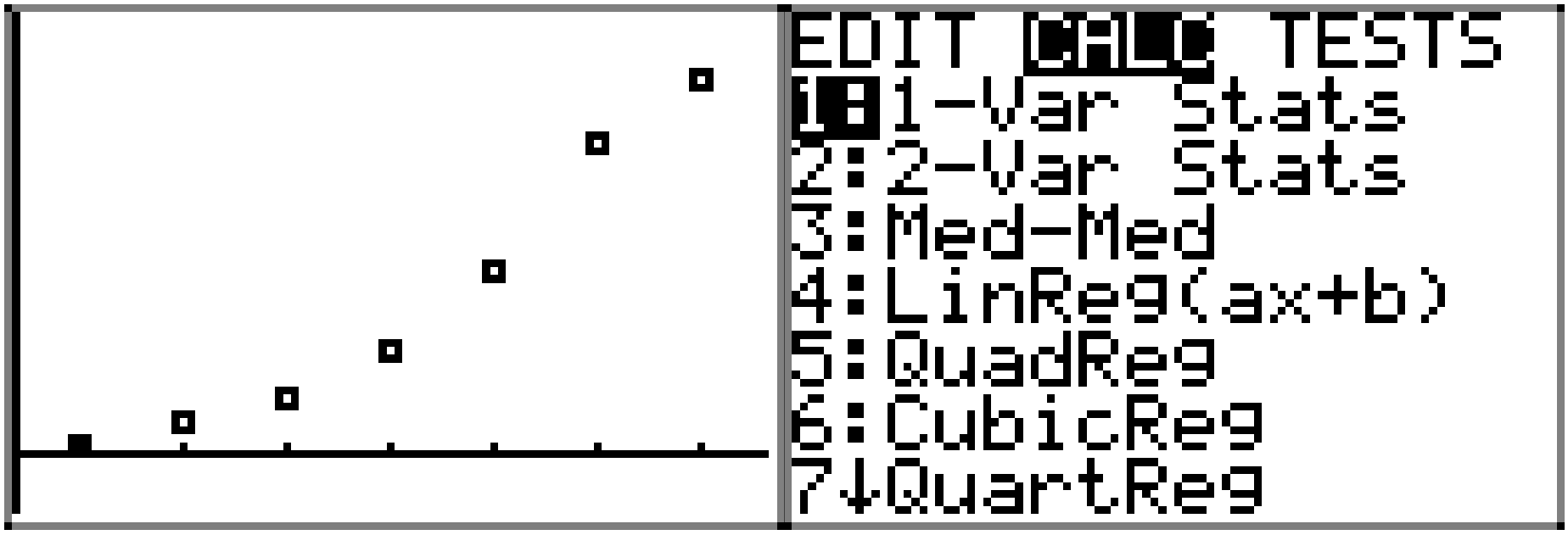


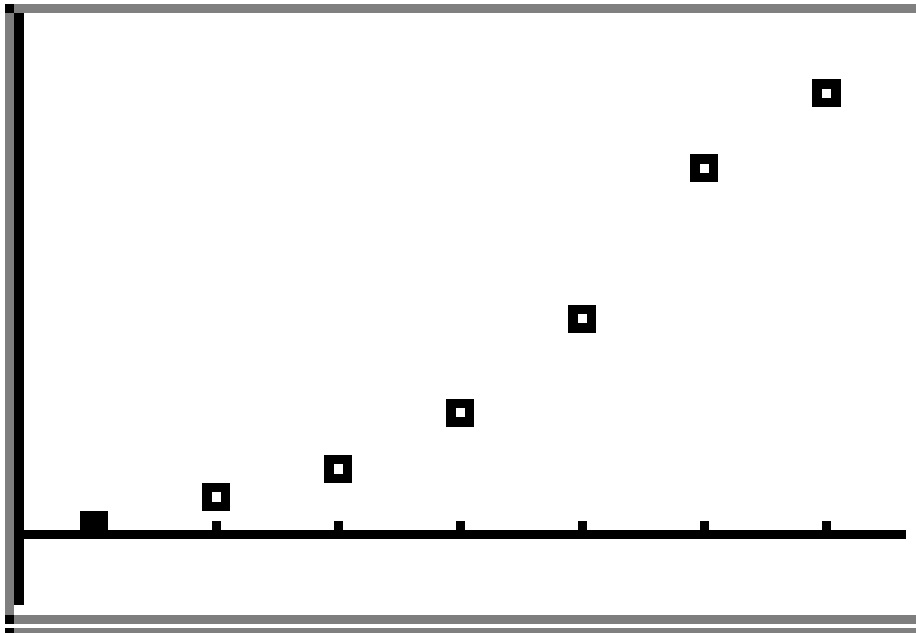
Make sure clear [Y=].



Make sure clear [Y=].



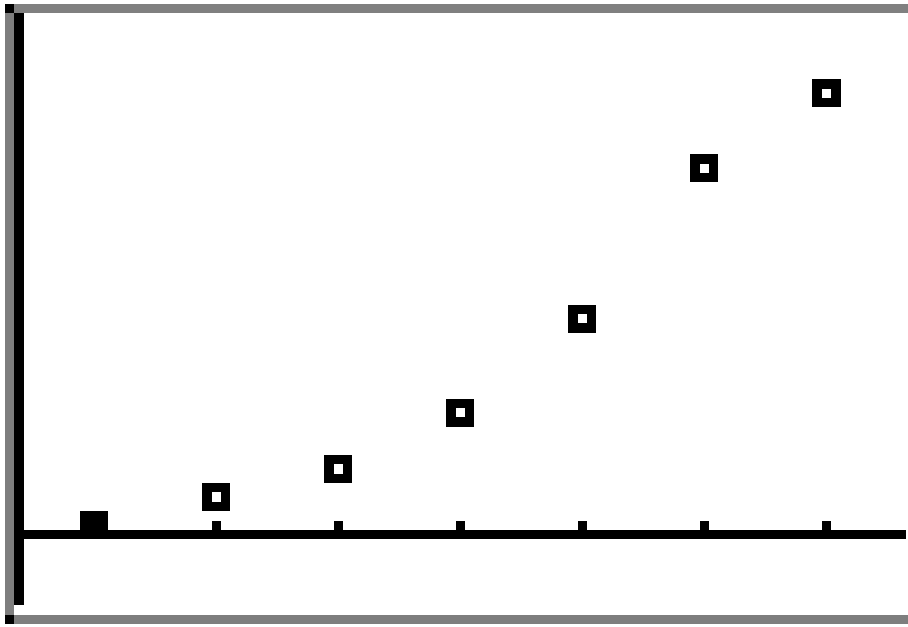
Make sure clear [Y=].



EDIT CALC TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg

Logistic
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:Y1
Calculate

Make sure clear [Y=].



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5:QuadReg
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7↓QuartReg

Logistic
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:Y1
Calculate

Logistic
y=c/(1+ae^(-bx))
a=147.5064091
b=.9300424746
c=185.9117027

$$S(x) = \frac{c}{1 + ae^{-bx}} \text{ million units.}$$

$a = 147.506,\ b = 0.930,\ c = 185.912$

$S(x) = \dfrac{c}{1 + ae^{-bx}}$ million units.

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(a') How many iPhone will be sold in 2014?

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(a') How many iPhone will be sold in 2014?

$S(8) = 171.1$ million

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(a') How many iPhone will be sold in 2014?

$S(8) = 171.1$ million

(b). What is the rate of change of the sales model?

$S(x) = \dfrac{c}{1 + ae^{-bx}}$ million units.

$a = 147.506,\ b = 0.930,\ c = 185.912$

(a') How many iPhone will be sold in 2014?

$S(8) = 171.1$ million

(b). What is the rate of change of the sales model?

$S'(x) = -c(1 + ae^{-bx})^{-2}(-abe^{-bx})$ million iPhones per year

where $a = 147.506,\ b = 0.930,\ c = 185.912$

(c). Using the model, calculate and interpret the rate of change of sales in 2014.

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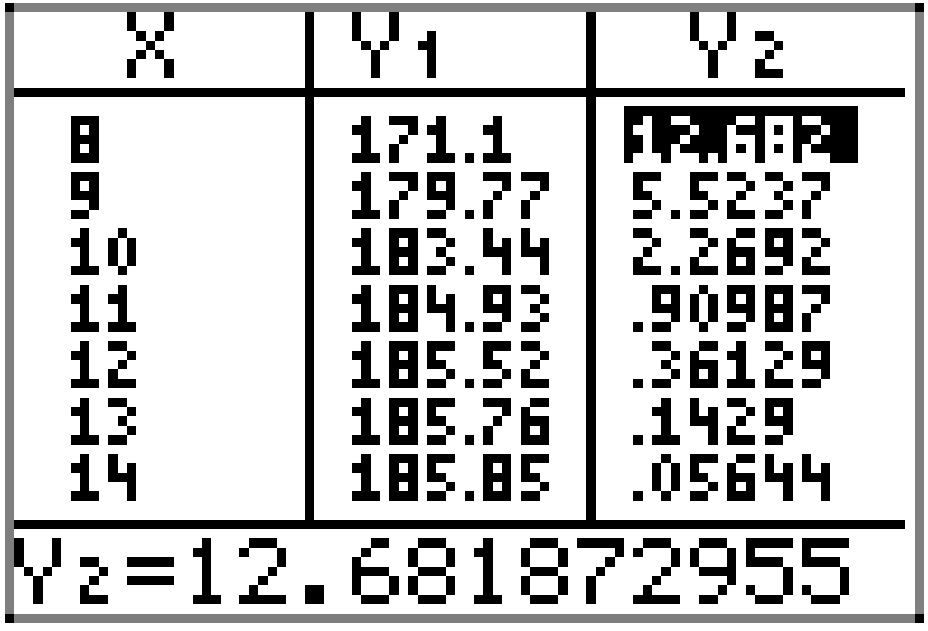

X	Y_1	Y_2
8	171.1	[redacted]
9	179.77	5.5237
10	183.44	2.2692
11	184.93	.90987
12	185.52	.36129
13	185.76	.1429
14	185.85	.05644

$Y_2 = 12.681872955$

(c). Using the model, calculate and interpret the rate of change of sales in 2014.

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$Y_2 = 12.681872955$

$S'(8) = 12.682$ millions units per year

(c). Using the model, calculate and interpret the rate of change of sales in 2014.

X	Y_1	Y_2
8	171.1	[redacted]
9	179.77	5.5237
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$Y_2 = 12.681872955$

$S'(8) = 12.682$ millions units per year

From 2014 to 2015, the sale of iPhone will increase by approx 12.682 million.

Example4

The following table gives the number of chocolate bars produced at a chocolate factory per number of Oompa-Loompas employed. Show work and give units for each answer.

$x = \#$ of Oompa-Loompas	5	10	15	20	25	30	35	40	45
Number of chocolate bars	12	18	20	20	17	15	15	18	26

(a) Let x stand for the number of Oompa-Loompas employed and let $B(x)$ stand for the number of chocolate bars produced. Fit a CUBIC MODEL to the data.

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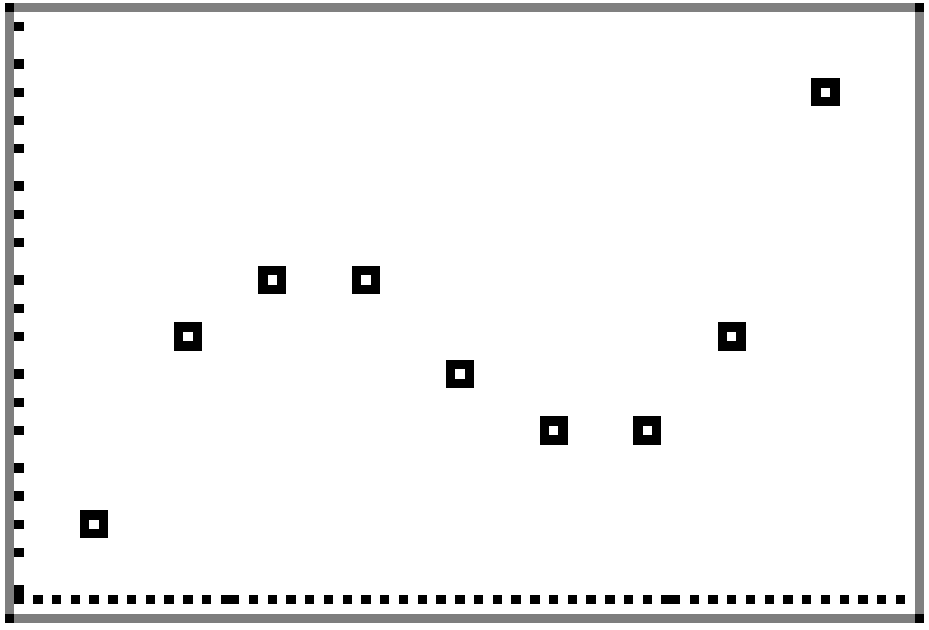
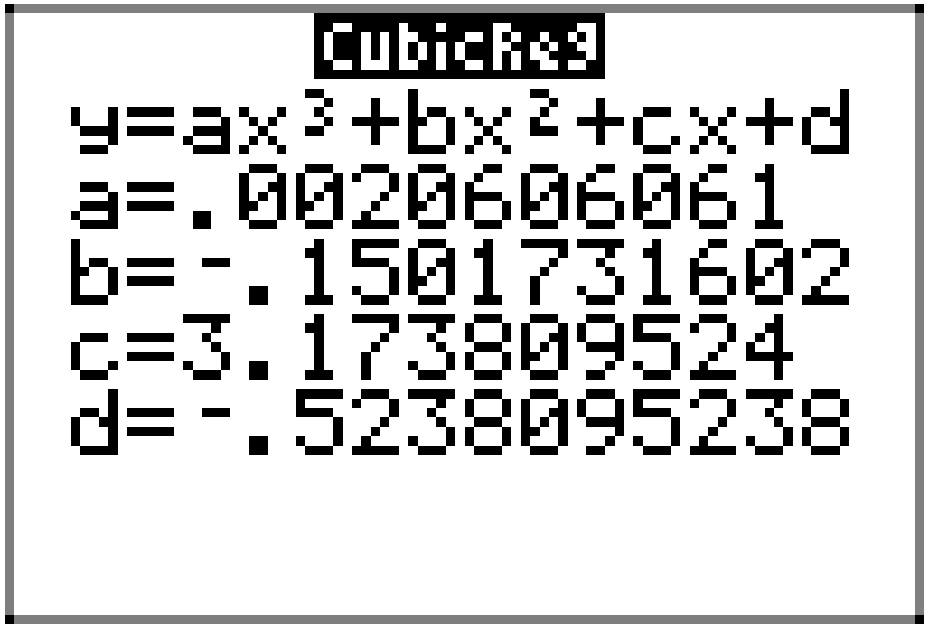
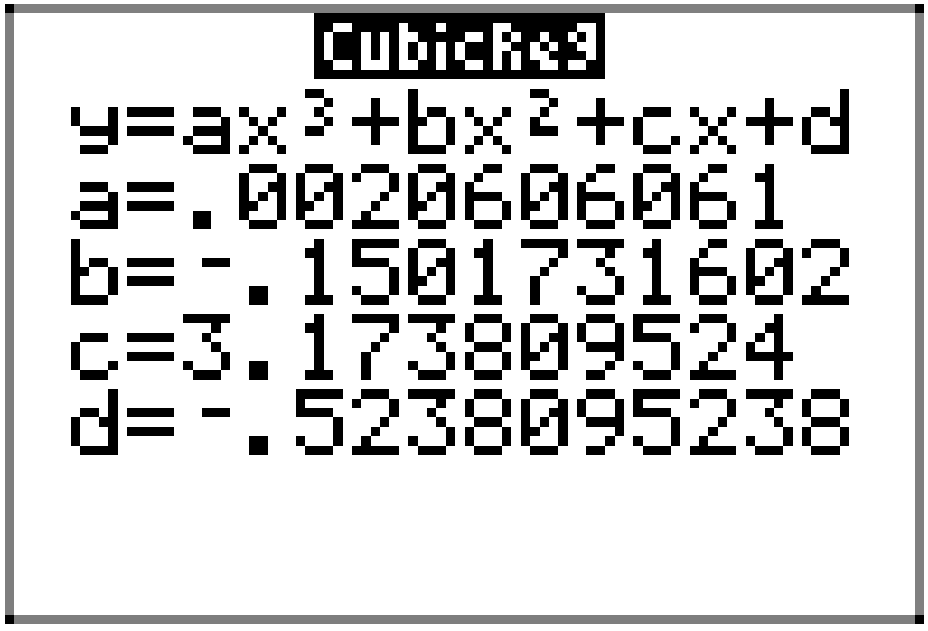


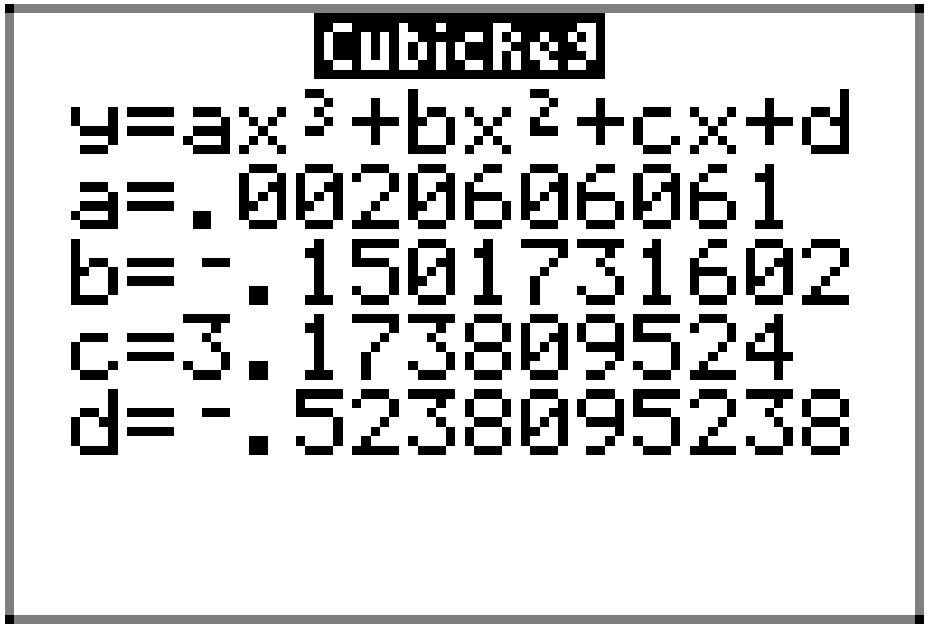

CubicReg
y=ax³+bx²+cx+d
a=.002060606061
b=-.1501731602
c=3.173809524
d=-.5238095238



$B(x) = ax^3 + bx^2 + cx + d$ chocolate bars

$a = 0.002,\ b = -0.150,\ c = 3.174,\ d = -0.524$

(b). What is the rate of change of the sales model?



$B(x) = ax^3 + bx^2 + cx + d$ chocolate bars

$a = 0.002$, $b = -0.150$, $c = 3.174$, $d = -0.524$

(b). What is the rate of change of the sales model?

$B'(x) = 0.006x^2 - 0.3x + 3.174$ chocolate bars/Oompa-Loompas.

(c). Using the model, calculate and interpret the rate of change of production when 11 Oompa-Loompas are employed.

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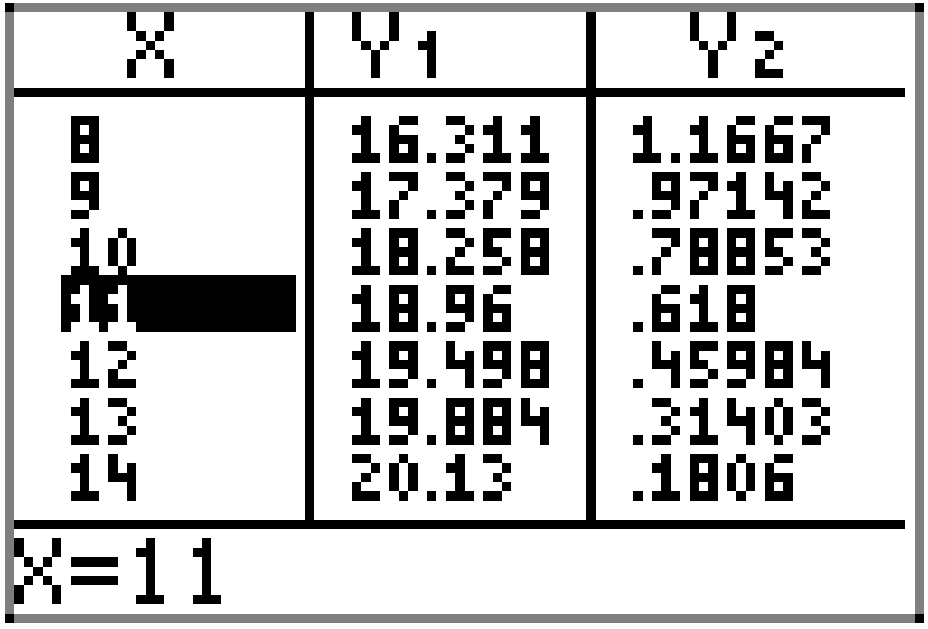
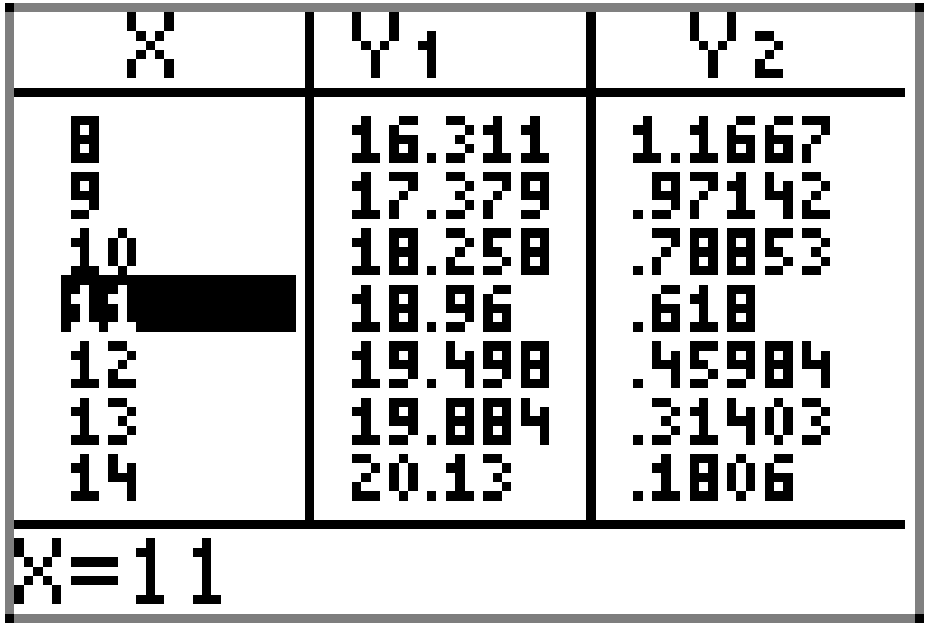


X	Y_1	Y_2
8	16.311	1.1667
9	17.379	.97142
10	18.258	.78853
11	18.96	.618
12	19.498	.45984
13	19.884	.31403
14	20.13	.1806

X=11

(c). Using the model, calculate and interpret the rate of change of production when 11 Oompa-Loompas are employed.



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$B'(11) = 0.618$ chocolate bars/Oompa-Loompas.

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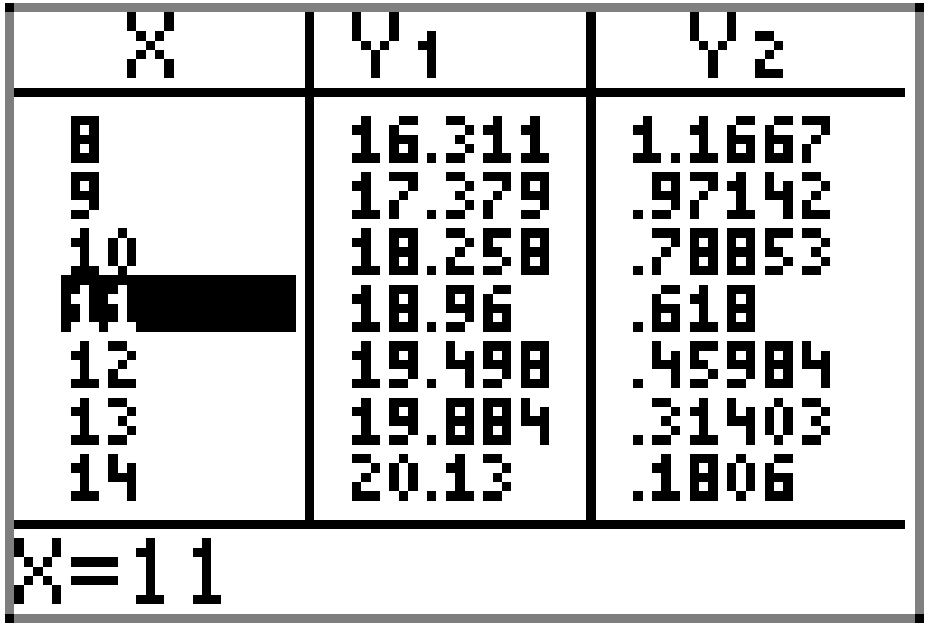

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X=11

$B'(11) = 0.618$ chocolate bars/Oompa-Loompas.

When the number of employed Oompa-Loompas increases from 11 to 12, then the production of chocolate bars increase by 0.618.

Example5 (Similar as HW19 in textbook 3.6.)

The profit from the supply of a certain commodity is modeled as

$$P(q) = 36qe^{-0.3q} \text{ dollars}$$

where q is the number of units produced.
(a). Write an expression for the rate of change of profit.

Example5 (Similar as HW19 in textbook 3.6.)

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$P'(q) = 36e^{-0.3q} + 36q(-0.3e^{-0.3q})$ dollars per unit

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(b). At what production level is the rate of change of profit zero?

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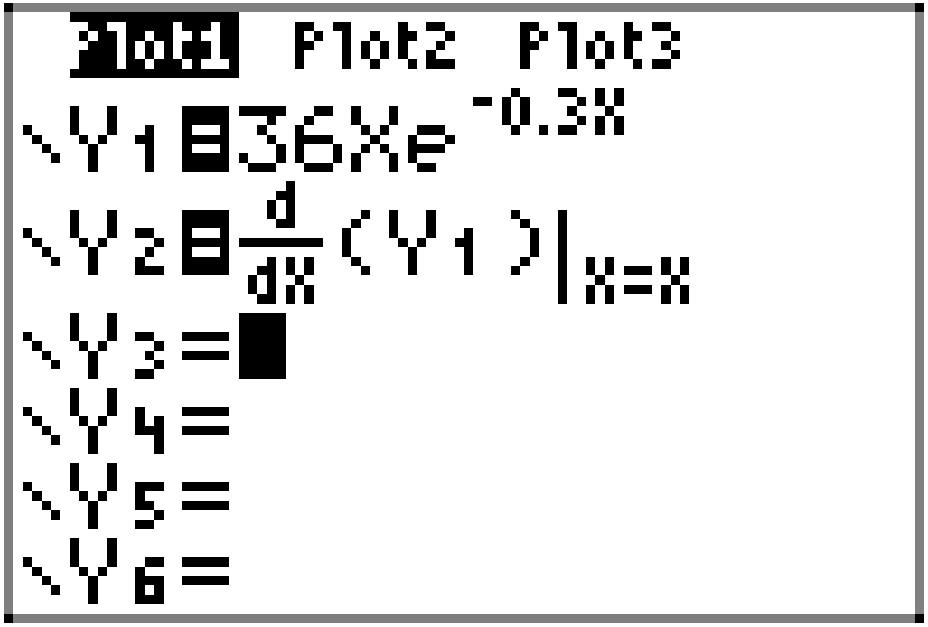
$P'(q) = 36e^{-0.3q} + 36q(-0.3e^{-0.3q})$ dollars per unit
(b). At what production level is the rate of change of profit zero?

Solve $P'(x) = 0$ by 2ed/calc/zero. $x = 3.333$

zoom 0:fit,


Plot1 Plot2 Plot3
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\Y3=
\Y4=
\Y5=
\Y6=

Plot1 Plot2 Plot3

$\backslash Y_1 = 36Xe^{-0.3X}$

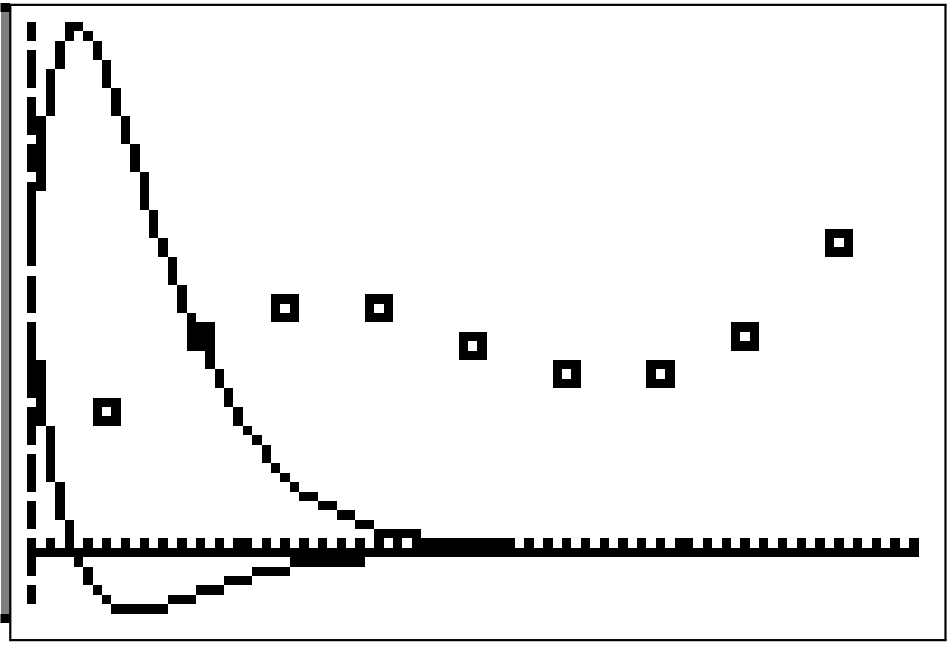
$\backslash Y_2 = \frac{d}{dX}(Y_1)|_{X=X}$

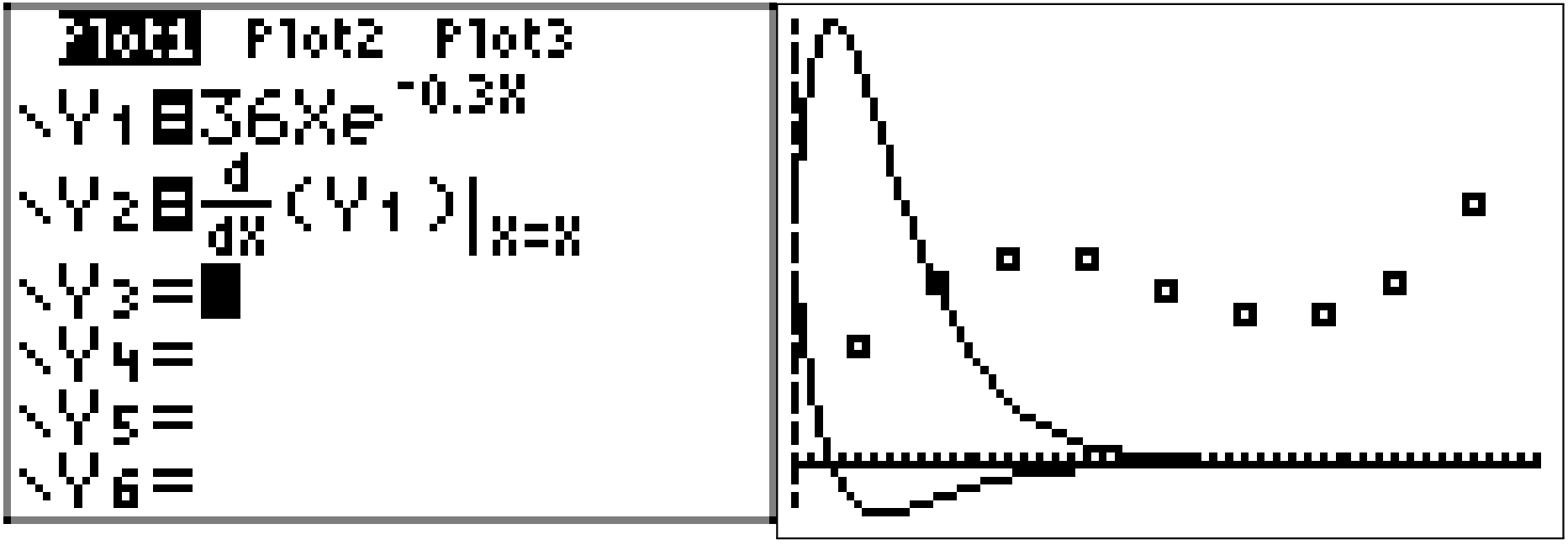
$\backslash Y_3 =$

$\backslash Y_4 =$

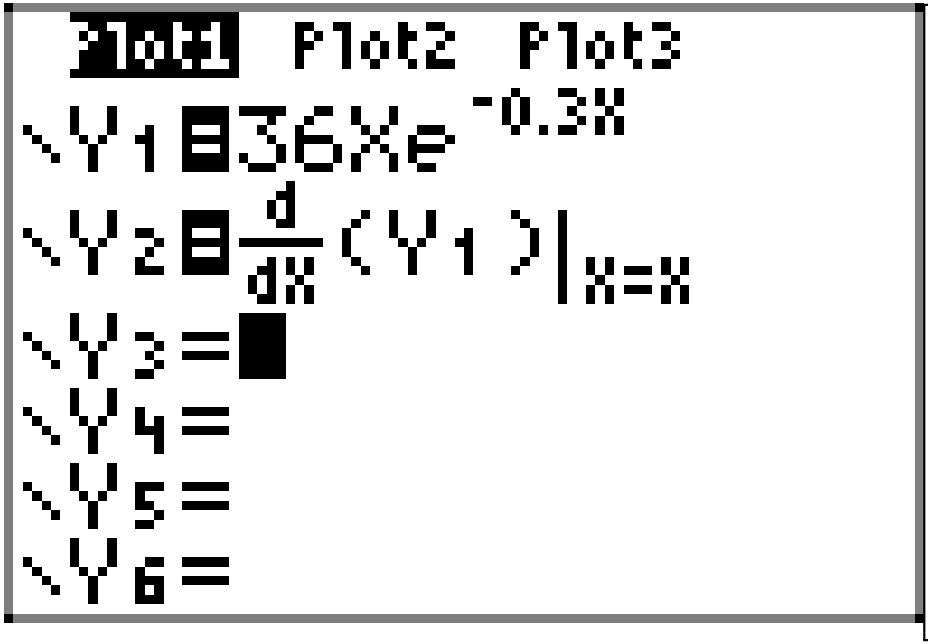
$\backslash Y_5 =$

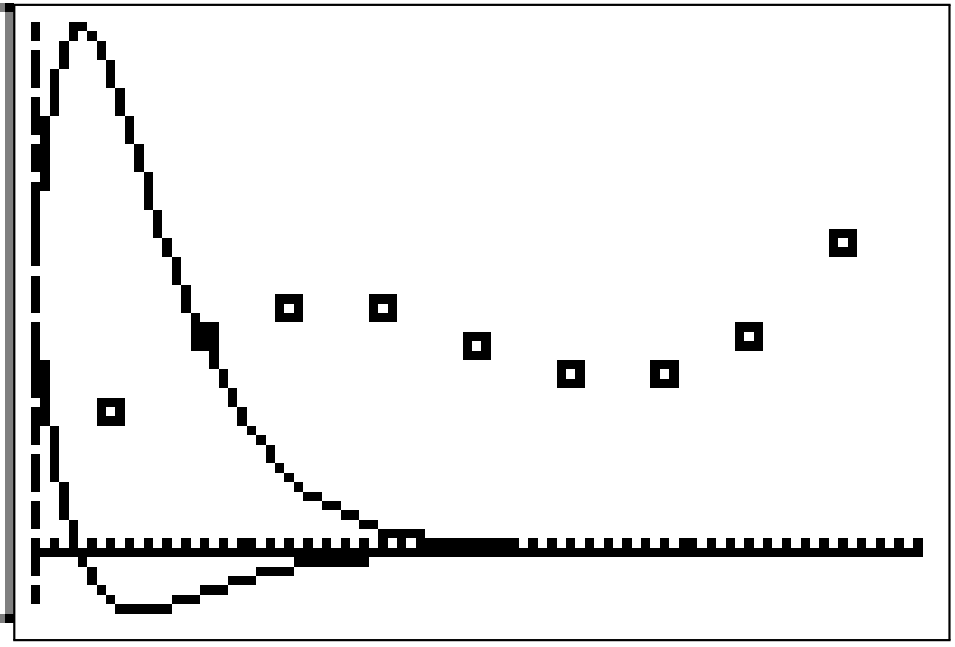
$\backslash Y_6 =$



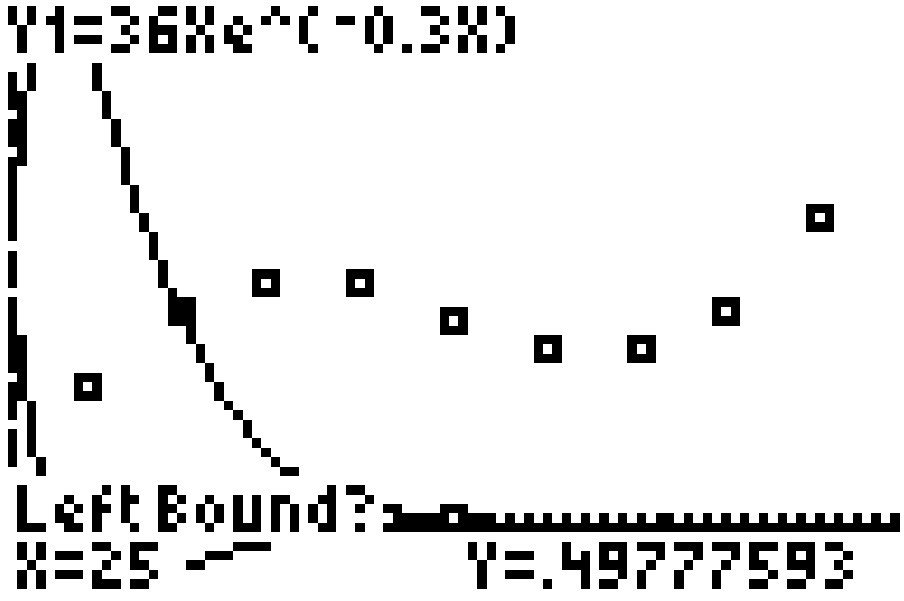


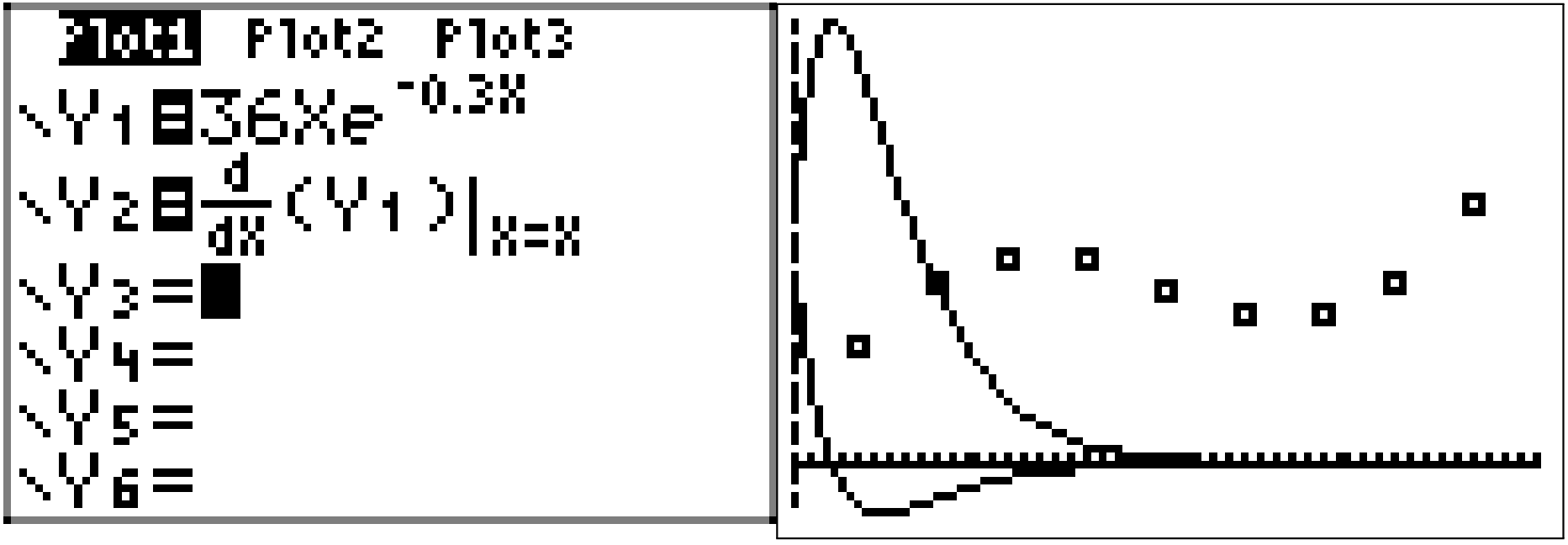
2ed/calc/zero



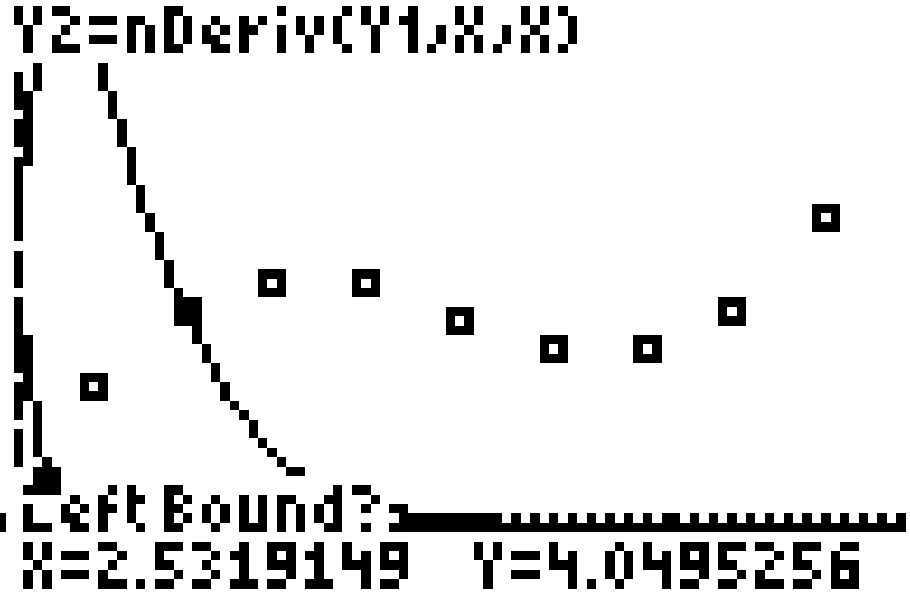


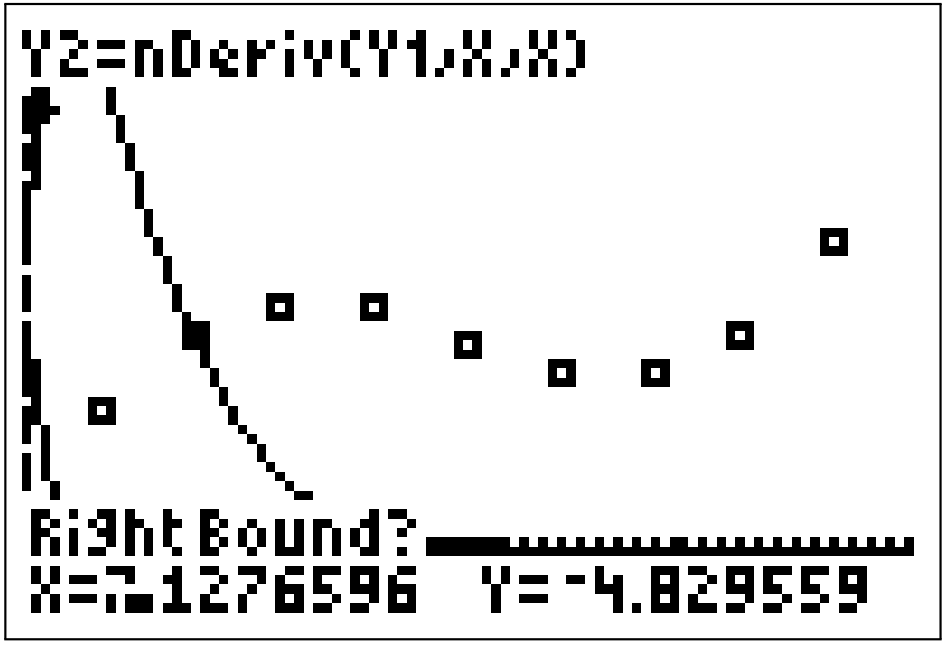
2ed/calc/zero

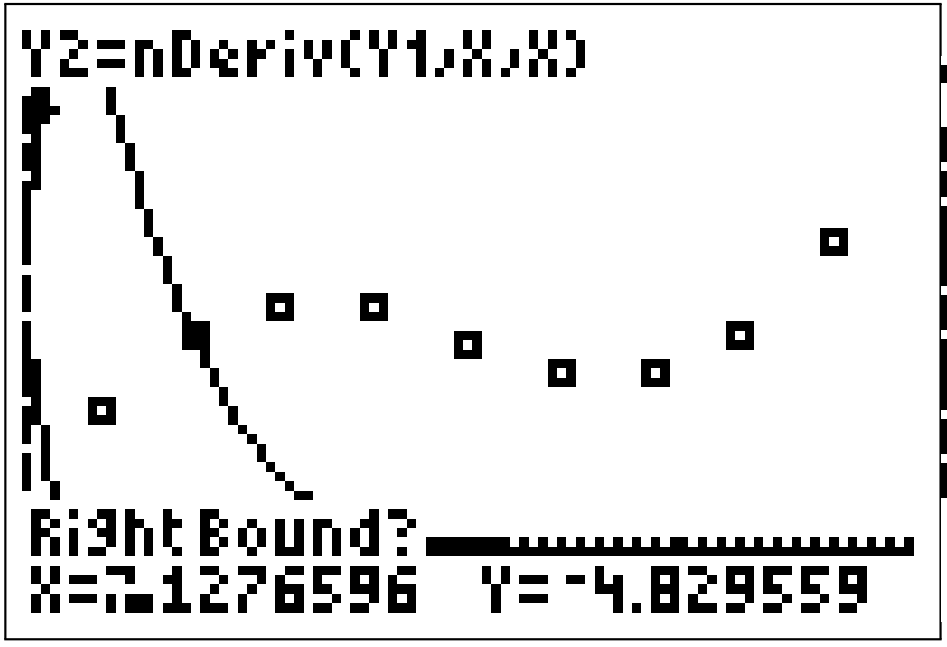


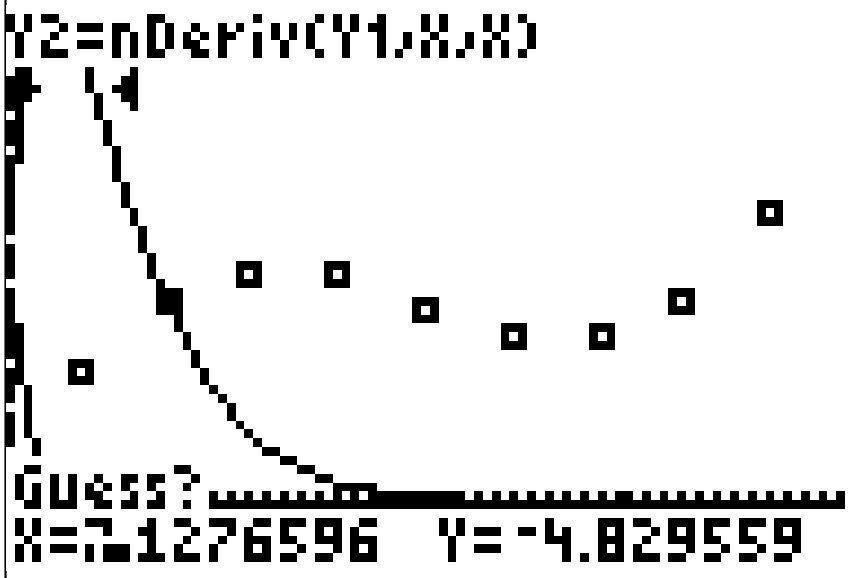


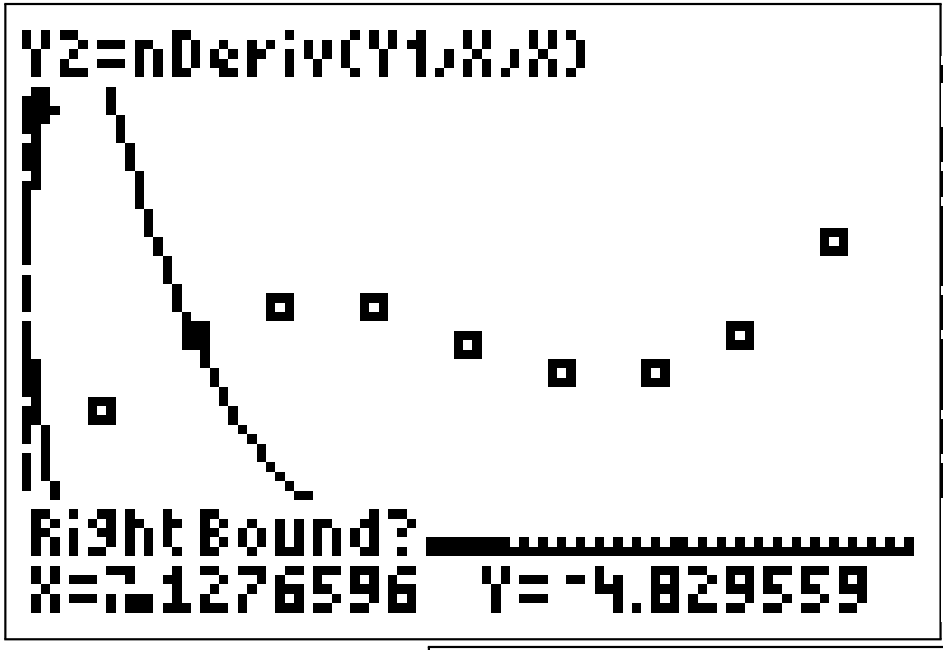
2ed/calc/zero

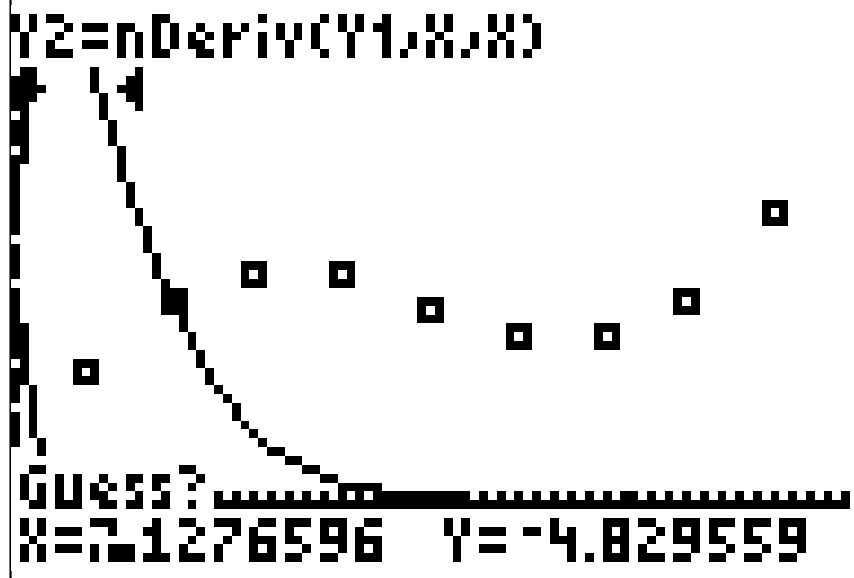


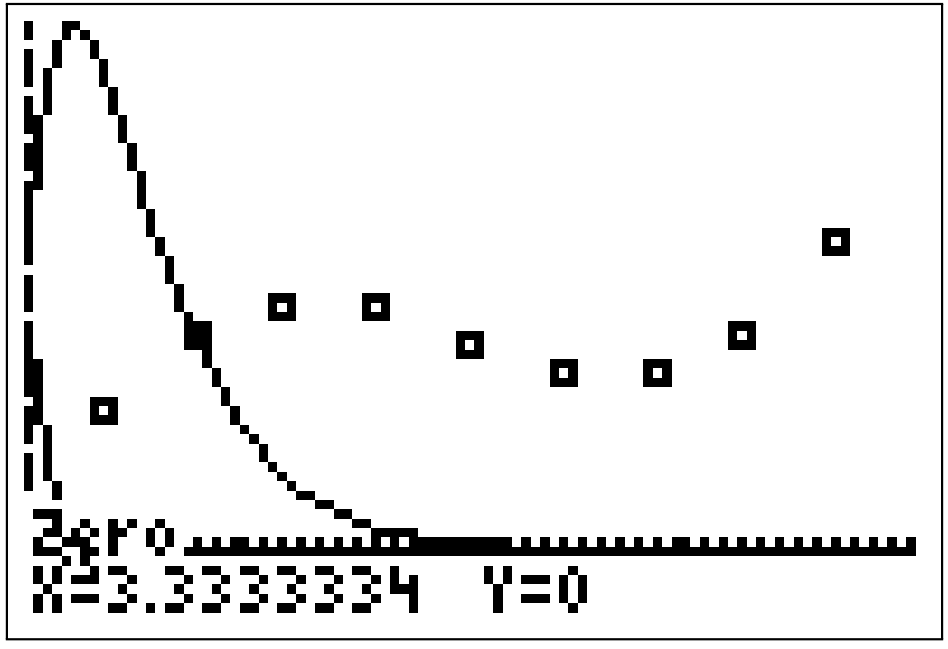

Y2=nDeriv(Y1,X,X)
RightBound?
X=7.1276596 Y=-4.829559


Y2=nDeriv(Y1,X,X)
RightBound?
X=7.1276596 Y=-4.829559


Y2=nDeriv(Y1,X,X)
Guess?
X=7.1276596 Y=-4.829559


Y2=nDeriv(Y1,X,X)
RightBound?
X=7.1276596 Y=-4.829559


Y2=nDeriv(Y1,X,X)
Guess?
X=7.1276596 Y=-4.829559


Zero
X=3.333334 Y=0

(c). What is profit at the production level found in part b?

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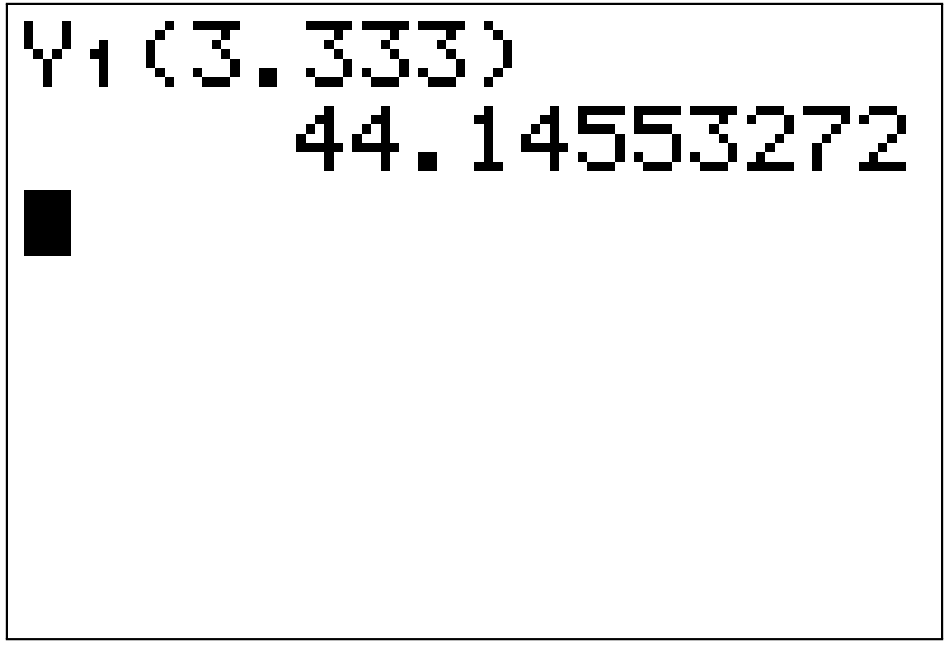
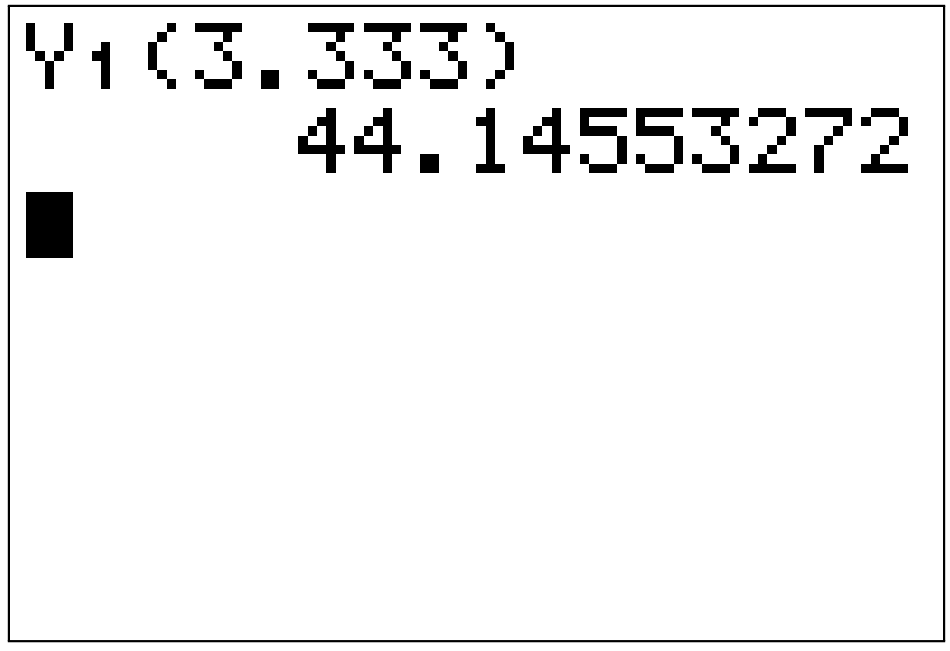


(c). What is profit at the production level found in part b?



$P(3.333) = Y1(3.333) = 44.146$

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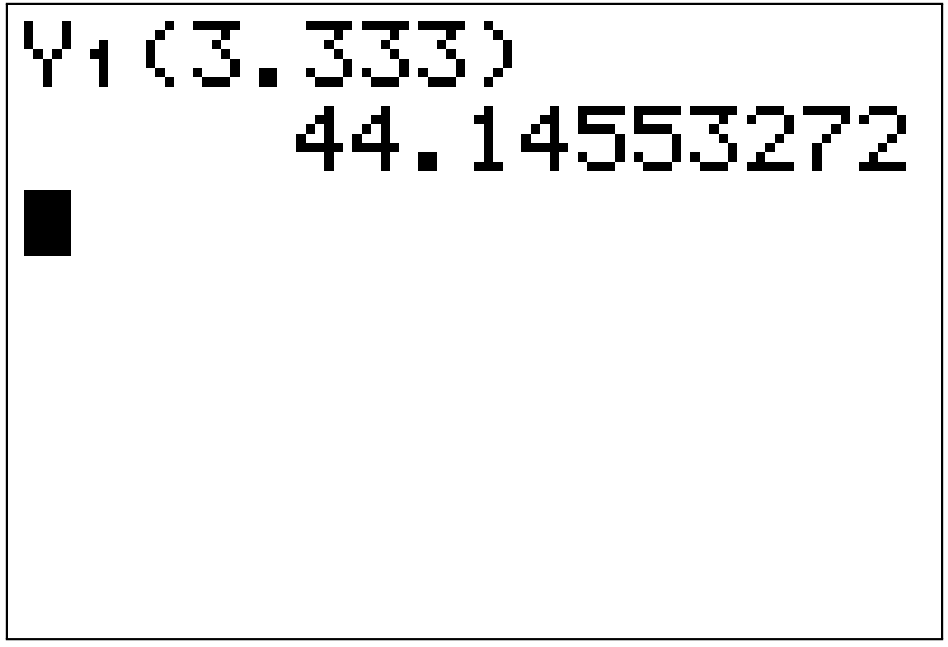

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