Math1231 Lecture 12

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1. Future value

1. Simple Interest

- Let P be the present value. (\$1000 for example)
- let r be the annual interest rate. (0.04 for example)
- The **accumulated interest** after t years is calculated as

$$I(t) = Prt \text{ dollars.}$$

- The **future value** at time t is

$$F_s(t) = P + Prt = P(1 + rt) \text{ dollars.}$$

2. Compound Interest

- Let P be the present value,
- let r be the annual percentage rate(APR).

The **future value** at time t in years of an investment (or loan) is

$$\boxed{F_c(t) = P \cdot \left(1 + \frac{r}{n}\right)^{nt} \text{ dollars.}}$$

where n is the number of compoundings per year.

- Compounded annually. $n = 1$.
- Compounded quarterly. $n = 4$.
- Compounded monthly. $n = 12$.
- Compounded semi-annually. $n = 2$.
- Compounded daily. $n = 365$.

3. Continuously compound Interest

The **future value** at time t in years of an investment

$$\boxed{F_e(t) = P \cdot e^{rt} \text{ dollars.}}$$

Example1(a). Write models for the future value of $1000 at 4% APR, for simple interest, compound annually, compound monthly, compound daily, compound continuously.

- Y1= $F_s(x) = 1000(1 + 0.04x)$
- Y2=$1000(1 + \frac{0.04}{1})^x$
- Y3=$1000(1 + \frac{0.04}{12})^{12x}$
- Y4=$1000(1 + \frac{0.04}{365})^{365x}$
- Y5=$F_e(x) = 1000e^{0.04x}$

Put the above functions to the calculator. Press [Y=], Enter functions, then using [TABLE].

Plot1 Plot2 Plot3

$\backslash Y_1 = 1000(1+0.04)$ ▸

$\backslash Y_2 = 1000(1+0.04$ ▸

$\backslash Y_3 = 1000(1+0.04$ ▸

$\backslash Y_4 = 1000(1+0.04$ ▸

$\backslash Y_5 = 1000e^{(0.04X)}$

X	Y_1	Y_2
1	1040	1040
2	1080	1081.6
3	1120	1124.9
4	1160	1169.9
5	1200	1216.7
6	1240	1265.3
7	1280	1315.9

$Y_1=1040$

X	Y_2	Y_3
1	1040	1040.7
2	1081.6	1083.1
3	1124.9	1127.3
4	1169.9	1173.2
5	1216.7	1221
6	1265.3	1270.7
7	1315.9	1322.5

$Y_2=1040$

X	Y_4	Y_5
1	1040.8	1040.8
2	1083.3	1083.3
3	1127.5	1127.5
4	1173.5	1173.5
5	1221.4	1221.4
6	1271.2	1271.2
7	1323.1	1323.1

$Y_5=1040.81077419$

Summarize the table data above: (future value of $1000 at 4% APR)

Year	Value(F_s)	F_c (n=1)	$F_c(n = 12)$	$F_c(n = 365)$	Value(F_e)
0	$1000	$1000	$1000	$1000	$1000
1	$1040	$1040	$1040.7	$1040.80	$1040.81
2	$1080	$1081.6	$1083.1	$1083.28	$1083.29
3	$1120	$1124.9	$1127.3	$1127.49	$1127.50
4	$1160	$1169.9	$1173.2	$1173.50	$1173.51
5	$1200	$1216.7	$1121.0	$1221.39	$1121.40

Example (b).

Write the **rate of change** equation for Each future value.

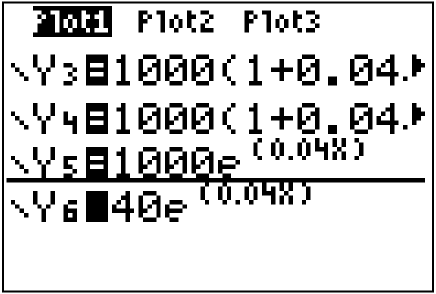
- Y1$=F_s(x) = 1000(1 + 0.04x)$
- $F_s'(x) = 1000(0.04) = 40$
- Y2$=1000(1 + \frac{0.04}{1})^x$
    - $(Y2)' = 1000(1 + \frac{0.04}{1})^x \ln(1.04)$
- Y3$=1000(1 + \frac{0.04}{12})^{12x}$
    - $(Y3)' = 1000(1 + \frac{0.04}{12})^{12x} \ln(1 + \frac{0.04}{12})12$
- Y4$=1000(1 + \frac{0.04}{365})^{365x}$
    - $(Y4)' = 1000(1 + \frac{0.04}{365})^{365x} \ln(1 + \frac{0.04}{365})365$
- Y5$=F_e(x) = 1000e^{0.04x}$
- $F_e'(x) = 40e^{0.04x}$

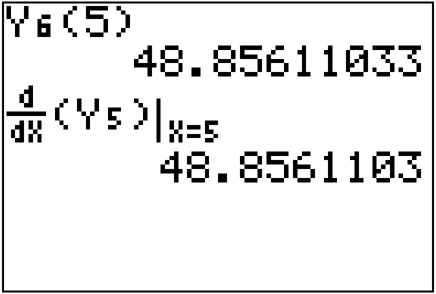
Example1 (c). How **quickly(rapidly)** is the investment $F_e(x)$ growing after 5 years.

Method1: Put $F_e'(x) = 40e^{0.04x}$ in the Calculator Y6. Then calculate Y6(5)=48.856 \$ per year.

Method2: Go back to home screen. Using [MATH], 8:[nDeriv(]

$$\frac{d}{dX}(Y5)\bigg|_{X=5} = 48.856 \text{ \$ per year.}$$

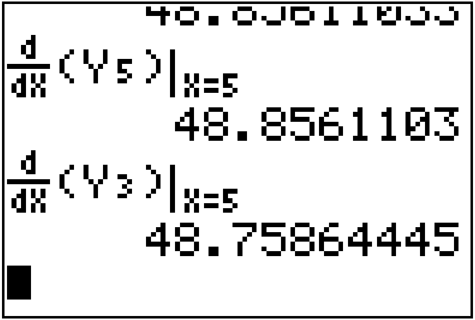




Example1 (d). How **quickly(rapidly)** is the investment $Y3$ growing after 5 years.

Using [MATH], 8:[nDeriv(]

$$\frac{d}{dX}(Y3)\bigg|_{X=5} = 48.759 \text{ \$ per year.}$$



Example. The number of phones in a country for the years 1990 through 2020 can be modeled by

$$N(x) = 1.1x^3 + 0.3x^2 + 0.2x + 5.11 \qquad \text{million phones,}$$

where x is the number of years after 1990.

(a). Write out the rate-of-change formula for the number of phones in the country.

$$N'(x) = 3.3x^2 + 0.6x + 0.2 \qquad \text{million phone per year}$$

(b). Fill in the following table.

	1991	2000	2010	Units
x				
number of phones				
Rate of change of $N(x)$				

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(b). Fill in the following table.

	1991	2000	2010	Units
x	1	10	20	year
number of phones				
Rate of change of $N(x)$				

Enter $N(x)$ to Y1, Enter $N'(x)$ to Y2. Check the [TABLE]

	1991	2000	2010	Units
x	1	10	20	year
number of phones	6.71	1137.11	8929.11	million
Rate of change of $N(x)$	4.1	336.2	1332.2	million per year

Another method find the rate of change using [nDeriv( ] Enter

$\frac{d}{dX}(Y1)\Big|_{X=X}$ to Y3