Exericse Compute the derivative of the following functions.

product 1. $f(x) = (3x^{10} - 5e^8)(3e^{2x} - 5(4^x))$.

$3x^{10}-5e^{8}$ $\quad$ $30x^{9}-0$

$3e^{2x}-5(4^{x})$ $\quad$ $6e^{2x}-5(\ln 4)4^{x}$

$f'(x)=(3x^{10}-5e^{8})(6e^{2x}-5(\ln 4)(4^{x}))+(3e^{2x}-5(4^{x}))(30x^{9})$

2. $f(x) = 22\ln\left(\frac{2}{3x} - 9x^{1.1}\right)$ $= 22\ln\left(\frac{2}{3}x^{-1}-9x^{1.1}\right)$

$u(x)=\frac{2}{3}x^{-1}-9x^{1.1}$ $\quad$ $u'(x)=-\frac{2}{3}x^{-2}-9.9x^{0.1}$

$g(u)=22\ln u$ $\quad$ $g'(u)=\frac{22}{u}$

$f'(x)=g'(u)\,u'(x)=\frac{22}{\frac{2}{3x}-9x^{1.1}}\left(-\frac{2}{3}x^{-2}-9.9x^{0.1}\right)$

product 3. $h(x) = \frac{3x - 2^x}{2\sqrt[3]{x^5}}$. $=(3x-2^{x})\left(\frac{1}{2}x^{-\frac{5}{3}}\right)$

$3x-2^{x}$ $\quad$ $3-(\ln 2)2^{x}$

$\frac{1}{2}x^{-\frac{5}{3}}$ $\quad$ $\frac{1}{2}\left(-\frac{5}{3}\right)x^{-\frac{8}{3}}$

$h'(x)=(3x-2^{x})\left(\frac{1}{2}\left(-\frac{5}{3}\right)x^{-\frac{8}{3}}\right)+\left(\frac{1}{2}x^{-\frac{5}{3}}\right)(3-(\ln 2)2^{x})$

4. $f(x) = \frac{3}{2(4x^5 - \sqrt[3]{x^4})^4} + x^5$

$u=4x^{5}-x^{-\frac{4}{3}}$ $\quad$ $u'(x)=20x^{4}+\frac{4}{3}x^{-\frac{7}{3}}$

$h(u)=\frac{3}{2u^{4}}=\frac{3}{2}u^{-4}$ $\quad$ $h'(u)=-6u^{-5}$

$f'(x)=-6\left(4x^{5}-x^{-\frac{4}{3}}\right)^{-5}\left(20x^{4}+\frac{4}{3}x^{-\frac{7}{3}}\right)$

produce

5. $h(x) = (2^x - 5x)(\sqrt[3]{x^4} - 3\ln x)$.

$2^x - 5x \qquad (\ln 2)2^x - 5$

$x^{\frac{4}{3}} - 3\ln x \qquad \frac{4}{3}x^{\frac{1}{3}} - \frac{3}{x}$

$$h'(x) = (2^x - 5x)\left(\frac{4}{3}x^{\frac{1}{3}} - \frac{3}{x}\right) + \left(x^{\frac{4}{3}} - 3\ln x\right)\left((\ln 2)2^x - 5\right)$$

6. $h(x) = \dfrac{21.45}{1 + 3.62e^{-2.1x}} + 10x$

$u(x) = 1 + 3.62e^{-2.1x} \qquad u'(x) = 3.62(-2.1)e^{-2.1x}$

$f(u) = \dfrac{21.45}{u} = 21.45u^{-1} \qquad f'(u) = -21.45u^{-2}$

$$h'(x) = -21.45(1 + 3.62e^{-2.1x})^{-2}\,3.62(-2.1)e^{-2.1x} + 10$$

product

7. $f(x) = \dfrac{3x^9 - 7(5^x)}{3e^{4x}} + 3x$. $= [3x^9 - 7(5^x)]\,\frac{1}{3}e^{-4x} \quad + 3x$

$3x^9 - 7(5^x) \qquad 27x^8 - 7(\ln 5)5^x$

$\frac{1}{3}e^{-4x} \qquad \frac{1}{3}(-4)e^{-4x}$

$$f'(x) = [3x^9 - 7(5^x)]\left(\frac{1}{3}(-4)e^{-4x}\right) + \left(\frac{1}{3}e^{-4x}\right)\left(27x^8 - 7(\ln 5)5^x\right)$$

8. $h(x) = 2e^{\sqrt[5]{x^2} + 3e^x} + \ln x$

$u = x^{\frac{2}{5}} + 3e^x \qquad u'(x) = \frac{2}{5}x^{-\frac{3}{5}} + 3e^x$

$f(u) = 2e^u \qquad f'(u) = 2e^u$

$$h'(x) = 2e^{x^{\frac{2}{5}} + 3e^x}\cdot\left(\frac{2}{5}x^{-\frac{3}{5}} + 3e^x\right) + \frac{1}{x}$$

Product 9. $g(x) = (3x^7 + 2(3^x))(e^{3.1x} - 5\ln x)$.

$3x^7+2(3^x) \quad \times \quad 21x^6+2(\ln 3)3^x$

$e^{3.1x}-5\ln x \quad\quad 3.1e^{3.1x}-\frac{5}{x}$

$$g'(x)=(3x^7+2(3^x))\left(3.1e^{3.1x}-\frac{5}{x}\right)+\left(e^{3.1x}-5\ln x\right)\left(21x^6+2(\ln 3)3^x\right)$$

10. $f(x) = \sqrt[3]{e^{3x} + 3\ln x + 2^x}$

$u=e^{3x}+3\ln x+2^x \qquad u'(x)=3e^{3x}+\frac{3}{x}+(\ln 2)2^x$

$g(u)=\sqrt[3]{u}=u^{\frac{1}{3}} \qquad g'(u)=\frac{1}{3}u^{-\frac{2}{3}}$

$$f'(x)=g'(u)\cdot u'(x)=\frac{1}{3}\left(e^{3x}+3\ln x+2^x\right)^{-\frac{2}{3}}\left(3e^{3x}+\frac{3}{x}+(\ln 2)2^x\right)$$

Product 11. $g(x) = (2x^5 + e^2)(2\ln x - 5x^6)$.

$2x^5+e^2 \quad \times \quad 10x^4+0$

$2\ln x-5x^5 \quad\quad \frac{2}{x}-25x^4$

$$g'(x)=(2x^5+e^2)\left(\frac{2}{x}-25x^4\right)+\left(2\ln x-5x^5\right)\left(10x^4\right)$$

12. $f(x) = \dfrac{e}{(4x^2 - \sqrt[5]{x^2})^2}$

$u=4x^2-x^{\frac{2}{5}} \qquad u'(x)=8x-\frac{2}{5}x^{-\frac{3}{5}}$

$g(u)=\frac{e}{u^2}=e\cdot u^{-2} \qquad g'(u)=-2e\,u^{-3}$

$$f'(x)=g'(u)\,u'(x)=-2e\left(4x^2-x^{\frac{2}{5}}\right)^{-3}\left(8x-\frac{2}{5}x^{-\frac{3}{5}}\right)$$

More Exercise: product + Chain Rule.

1. $f(x) = \big(2\ln(3x^5 + 3x)\big)(e^x + 4x^5)$

$g(x) = 2\ln(3x^5+3x)$ $\quad g'(x) = \dfrac{2(15x^4+3)}{3x^5+3x}$ ← Chain Rule.

$h(x) = e^x + 4x^5$ $\quad h'(x) = e^x + 20x^4$

$$f'(x) = 2\ln(3x^5+3x)\,(e^x+20x^4) + \frac{(e^x+4x^5)\,2\,(15x^4+3)}{3x^5+3x}$$

2. $h(x) = (8x^2 + e^3)\left(\dfrac{12}{1+5e^{-0.3x}}\right)$

$f(x) = 8x^2 + e^3$ $\quad f'(x) = 16x + 0$

$g(x) = 12(1+5e^{-0.3x})^{-1}$ $\quad g'(x) = -12(1+5e^{-0.3x})^{-2}(-1.5e^{-0.3x})$ ← chain Rule

$$h'(x) = (8x^2+e^3)\big((-12)(1+5e^{-0.3x})^{-2}(-1.5e^{-0.3x})\big) + 12(1+5e^{-0.3x})^{-1}\,16x$$

3. $f(x) = \dfrac{2x^3 + 3e}{(x^2 + 2x)^2} = (2x^3+3e)(x^2+2x)^{-2}$

$2x^3+3e$ $\quad 6x^2$

$(x^2+2x)^{-2}$ $\quad -2(x^2+2x)^{-3}(2x+2)$ ← Chain Rule

$$f'(x) = (2x^3+3e)\big(-2(x^2+2x)^{-3}(2x+2)\big) + (x^2+2x)^{-2}\,6x^2$$

4. $f(x) = 20(-2x^{-2} + 3e^{5x^3})^6$ Chain Rule twice

$$f'(x) = 120(-2x^{-2}+3e^{5x^3})^5(4x^{-3}+3e^{5x^3}(15x^2))$$

5. $f(x) = 5\ln(e^{3x^4} + 2x)$ Chain Rule twice

$$f'(x) = \frac{5}{e^{3x^4}+2x} \cdot (e^{3x^4} \cdot 12x^3 + 2)$$

6. $f(x) = \left(5e^{\sqrt{x}+2x^3}\right)\left(2.2(\ln x) + e^3\right)$

$5e^{x^{\frac{1}{2}}+2x^3}$ $\quad 5e^{x^{\frac{1}{2}}+2x^3}(\frac{1}{2}x^{-\frac{1}{2}}+6x^2)$ ← Chain Rule

$2.2\ln x + e^3$ $\quad \frac{2.2}{x}$

$$f'(x) = (5e^{x^{\frac{1}{2}}+2x^3})\left(\frac{2.2}{x}\right) + (2.2\ln x + e^3)\left(5e^{x^{\frac{1}{2}}+2x^3}\left(\frac{1}{2}x^{-\frac{1}{2}}+6x^2\right)\right)$$