

Example: Compute the derivative of $f(x) = (3x^{10} - 5x^8)(3e^x - 4^x)$.

$$\begin{array}{l} 3x^{10} - 5x^8 \\ 3e^x - 4^x \end{array} \times \begin{array}{l} 30x^9 - 40x^7 \\ 3e^x - (\ln 4)4^x \end{array}$$

$$f'(x) = (3x^{10} - 5x^8)(3e^x - (\ln 4)4^x) + (3e^x - 4^x)(30x^9 - 40x^7)$$

Example: Compute the derivative of $g(x) = (2x^5 + e^2)(2\ln x - 5x^6)$.

$$\begin{array}{l} 2x^5 + e^2 \\ 2\ln x - 5x^2 \end{array} \times \begin{array}{l} 10x^4 \\ \frac{2}{x} - 10x \end{array}$$

$$g'(x) = (2x^5 + e^2)\left(\frac{2}{x} - 10x\right) + (2\ln x - 5x^2)(10x^4)$$

Example: Compute the derivative of $h(x) = (2^x - 10)(\sqrt[3]{x^4} - 3\ln x)$.

$$\begin{array}{l} 2^x - 10 \\ x^{\frac{4}{3}} - 3\ln x \end{array} \times \begin{array}{l} (\ln 2)2^x \\ \frac{4}{3}x^{\frac{1}{3}} - \frac{3}{x} \end{array}$$

$$h'(x) = (2^x - 10)\left(\frac{4}{3}x^{\frac{1}{3}} - \frac{3}{x}\right) + (x^{\frac{4}{3}} - 3\ln x)(\ln 2)2^x$$

Example: Compute the derivative of $h(x) = \frac{3x^{1.2} - 2^x}{2\sqrt[3]{x^5}} = (3x^{1.2} - 2^x)\left(\frac{1}{2}x^{-\frac{5}{3}}\right)$

$$\begin{array}{l} 3x^{1.2} - 2^x \\ \frac{1}{2}x^{-\frac{5}{3}} \end{array} \times \begin{array}{l} 3.6x^{0.2} - (\ln 2)2^x \\ \frac{1}{2}\left(-\frac{5}{3}\right)x^{-\frac{8}{3}} \end{array}$$

$$h'(x) = (3x^{1.2} - 2^x)\left(\frac{1}{2}\left(-\frac{5}{3}\right)x^{-\frac{8}{3}}\right) + \left(\frac{1}{2}x^{-\frac{5}{3}}\right)(3.6x^{0.2} - (\ln 2)2^x)$$

Exercise 1. $g(x) = (3x^7 + e^3)(e^{3x} - 5\ln x)$.

$$\begin{array}{l} 3x^7 + e^3 \\ e^{3x} - 5\ln x \end{array} \times \begin{array}{l} 21x^6 + e^3 \\ 3e^{3x} - \frac{5}{x} \end{array}$$

$$g'(x) = (3x^7 + e^3)\left(3e^{3x} - \frac{5}{x}\right) + (e^{3x} - 5\ln x)(21x^6 + e^3)$$

Exercise 2. $f(x) = \frac{3x^9 - 7x^5}{3e^x} = (3x^9 - 7x^5)\left(\frac{1}{3}e^{-x}\right)$

$$\begin{array}{l} 3x^9 - 7x^5 \\ \frac{1}{3}e^{-x} \end{array} \times \begin{array}{l} 27x^8 - 35x^4 \\ -\frac{1}{3}e^{-x} \end{array}$$

$$f'(x) = (3x^9 - 7x^5)\left(-\frac{1}{3}e^{-x}\right) + \left(\frac{1}{3}e^{-x}\right)(27x^8 - 35x^4)$$

Question: What is the derivative of $h(x) = xe^x$?

- The product function of two functions $f(x)$ and $g(x)$ is defined by $(f \cdot g)(x) = f(x) \cdot g(x)$.

Product Rule for Derivative.

If $f(x)$ and $g(x)$ are differentiable, then

$$(f \cdot g)'(x) = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

$$\begin{array}{l} f \\ g \end{array} \times \begin{array}{l} f' \\ g' \end{array}$$

Example: Compute the derivative of $h(x) = xe^x$.

$$\begin{array}{l} f(x) = x \\ g(x) = e^x \end{array} \times \begin{array}{l} f'(x) = 1 \\ g'(x) = e^x \end{array} \quad h'(x) = xe^x + e^x$$

Example: Compute the derivative of $h(x) = (4x^3 - x^2)(5^x + 3e^x)$.

$$\begin{array}{l} f(x) = 4x^3 - x^2 \\ g(x) = 5^x + 3e^x \end{array} \times \begin{array}{l} f'(x) = 12x^2 - 2x \\ g'(x) = (\ln 5)5^x + 3e^x \end{array}$$

$$h'(x) = (4x^3 - x^2)((\ln 5)5^x + 3e^x) + (5^x + 3e^x)(12x^2 - 2x)$$

Example: Compute the derivative of $f(x) = (3.4x^{2.1} + 2e^{2x})(\ln x - 3x)$.

$$\begin{array}{l} g(x) = 3.4x^{2.1} + 2e^{2x} \\ h(x) = \ln x - 3x \end{array} \quad \begin{array}{l} g'(x) = 3.4 \times 2.1 x^{1.1} + 4e^{2x} \\ h'(x) = \frac{1}{x} - 3 \end{array}$$

$$f'(x) = (3.4x^{2.1} + 2e^{2x})\left(\frac{1}{x} - 3\right) + (\ln x - 3x)(3.4 \times 2.1 x^{1.1} + 4e^{2x})$$

Example: Compute the derivative of $h(x) = \frac{3 \ln x - 2e^x}{4\sqrt[5]{x^2}}$.

$$h(x) = (3 \ln x - 2e^x) \left(\frac{1}{4} x^{-\frac{2}{5}}\right)$$

$$\begin{array}{l} 3 \ln x - 2e^x \\ \frac{1}{4} x^{-\frac{2}{5}} \end{array} \times \begin{array}{l} \frac{3}{x} - 2e^x \\ \frac{1}{4} \left(-\frac{2}{5}\right) x^{-\frac{7}{5}} \end{array}$$

$$h'(x) = (3 \ln x - 2e^x) \left(\frac{1}{4} \left(-\frac{2}{5}\right) x^{-\frac{7}{5}}\right) + \left(\frac{1}{4} x^{-\frac{2}{5}}\right) \left(\frac{3}{x} - 2e^x\right)$$